Perspectives on Tail Risk Hedging

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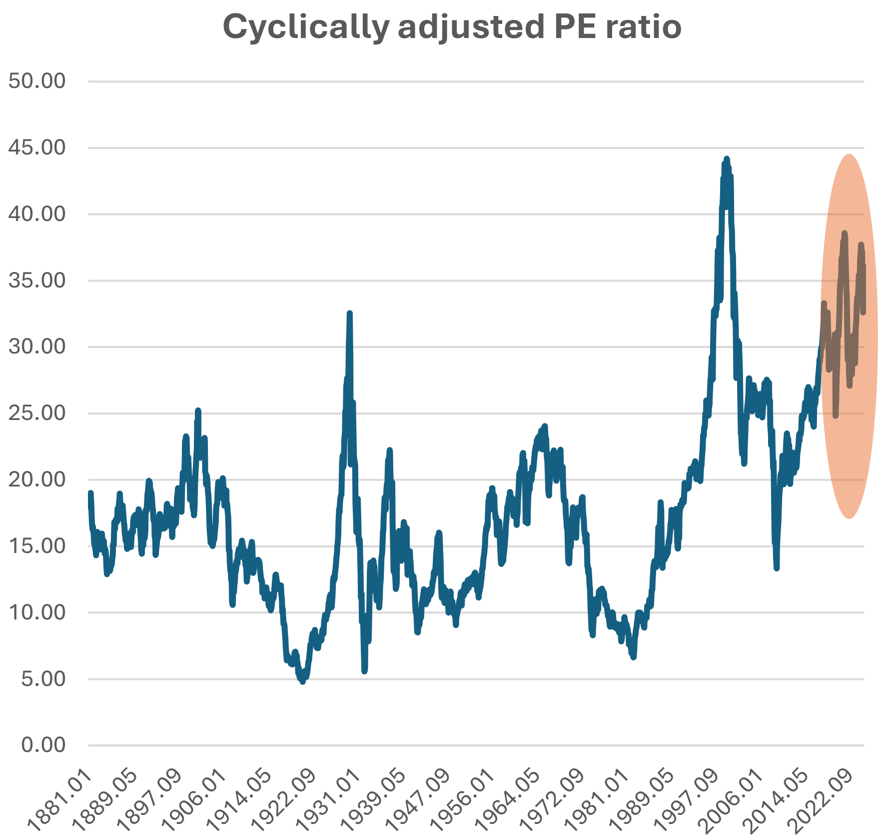
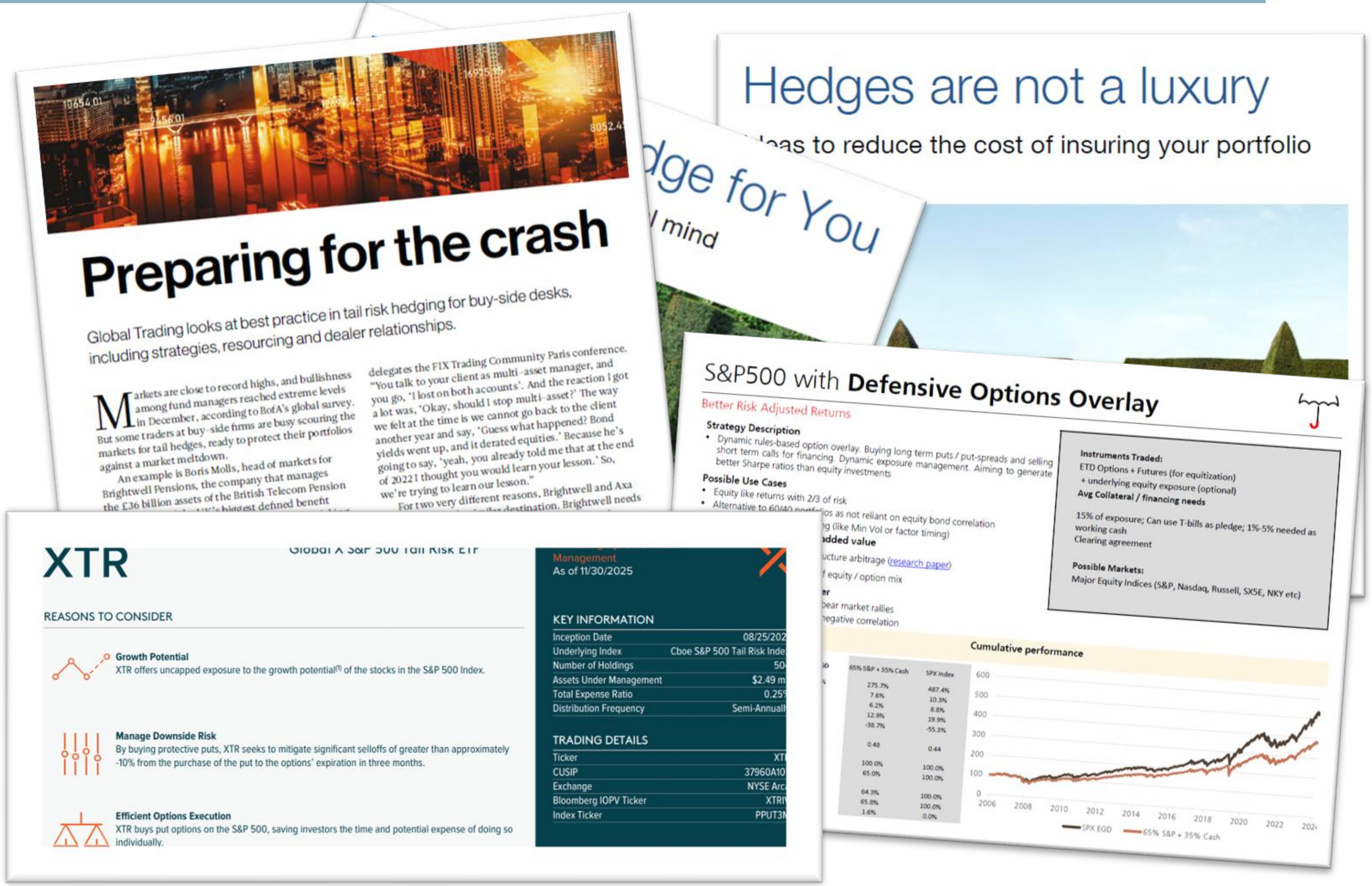
Based on: „**Carry, Convexity and Reliability – Navigating Tail Risk Hedging Dilemmas”** (w. Chris Marais & Bruno Schwalbach)

With the equity market in „bubble territory” should we be putting on some *tail risk hedges*?

Tail risk hedging can be loosely defined as a strategy designed to generate outsized returns during rare but highly impactful events, characteristic of the “tail” of the returns distribution.



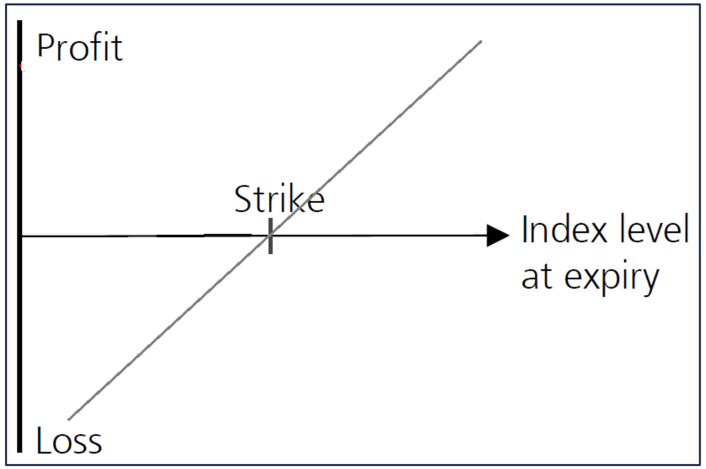


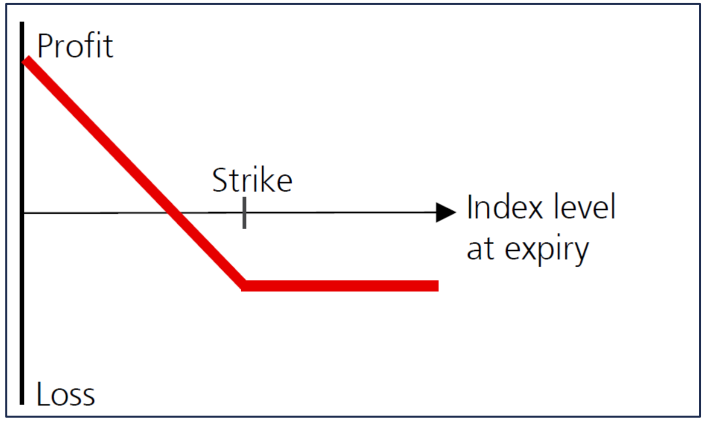
Option-based tail risk hedging looks straightforward in theory

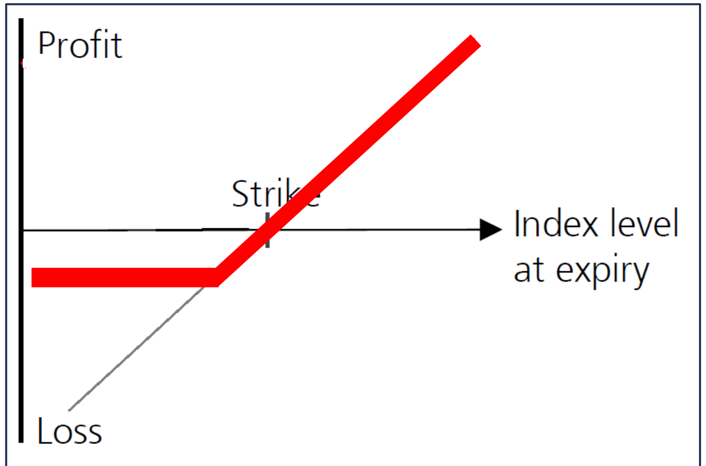
1. Start with uncovered (long) equity exposure
2. Add a put option (right but no obligation to sell underlying at pre-agreed price)
3. Arrive at a combination with limited downside and potentially unlimited upside

...but can be non-trivial in practice

- Many design choices give rise to difficult trade-offs
- Protection is costly and puts are expected to (mostly) expire worthless – does hedging make sense for a long-term investor?
- Will hedges actually work when you need them most?







The test of 2022: did put protection work?

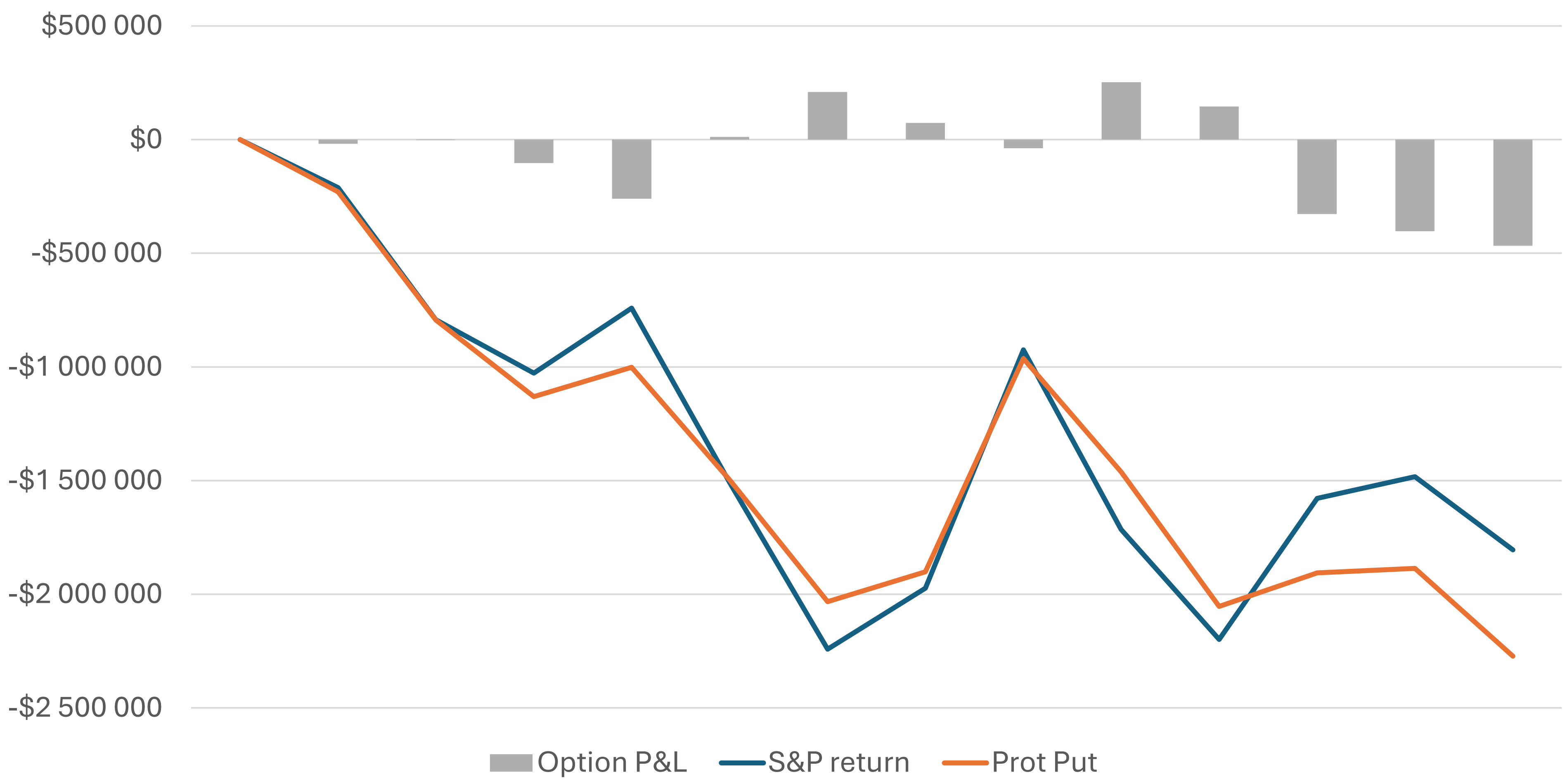
- Start with $10 million, buy S&P 500 and **<u>roll 5% OTM puts every quarter</u>**
- **Example 1**: on 31-12-2021 buy S&P 500, and ~21 SPX puts (US 03/18/22 P4530)

	S&P (ret)	Dividends	S&P (P&L)	Premium cost	Exercise P&L
12/31/2021 - 03/17/2022	-7.44%	30 147.60	- 713 655.00	159 667.00	258 278.00
03/17/2022 - 06/16/2022	-16.9%	34 708.00	- 1 528 179.00	267 720.00	1 108 225.00
06/16/2022 - 09/15/2022	6.4%	34 621.00	526 797.00	292 897.00	734.00
09/15/2022 - 12/15/2022	-0.1%	34 739.00	22 990.00	237 507.00	378 920.00
Total	-18.3%	138 871.00	- 1 692 047.00	957 791.00	1 746 157.00

	S&P return	Option P&L	Prot Put
12/31/2021 - 03/17/2022	- 713 655.00	98611	- 615 044.00
03/17/2022 - 06/16/2022	- 2 241 834.00	939116	- 1 302 718.00
06/16/2022 - 09/15/2022	- 1 715 037.00	646953	- 1 068 084.00
09/15/2022 - 12/15/2022	- 1 692 047.00	788366	- 903 681.00

The test of 2022: did put protection work?

- Start with $10 million, buy S&P 500 and roll 5% OTM puts
- **Example 2:** on 31-12-2021 buy S&P 500, and roll **monthly** SPX puts



The problems

- Is there a place for a negatively-yielding asset (strategic hedge) in a portfolio?
- What are the key design choices and trade-offs to navigate?
- How do we come up with a thoughtful program without over-relying on backtests (overfit to very few occurences)?

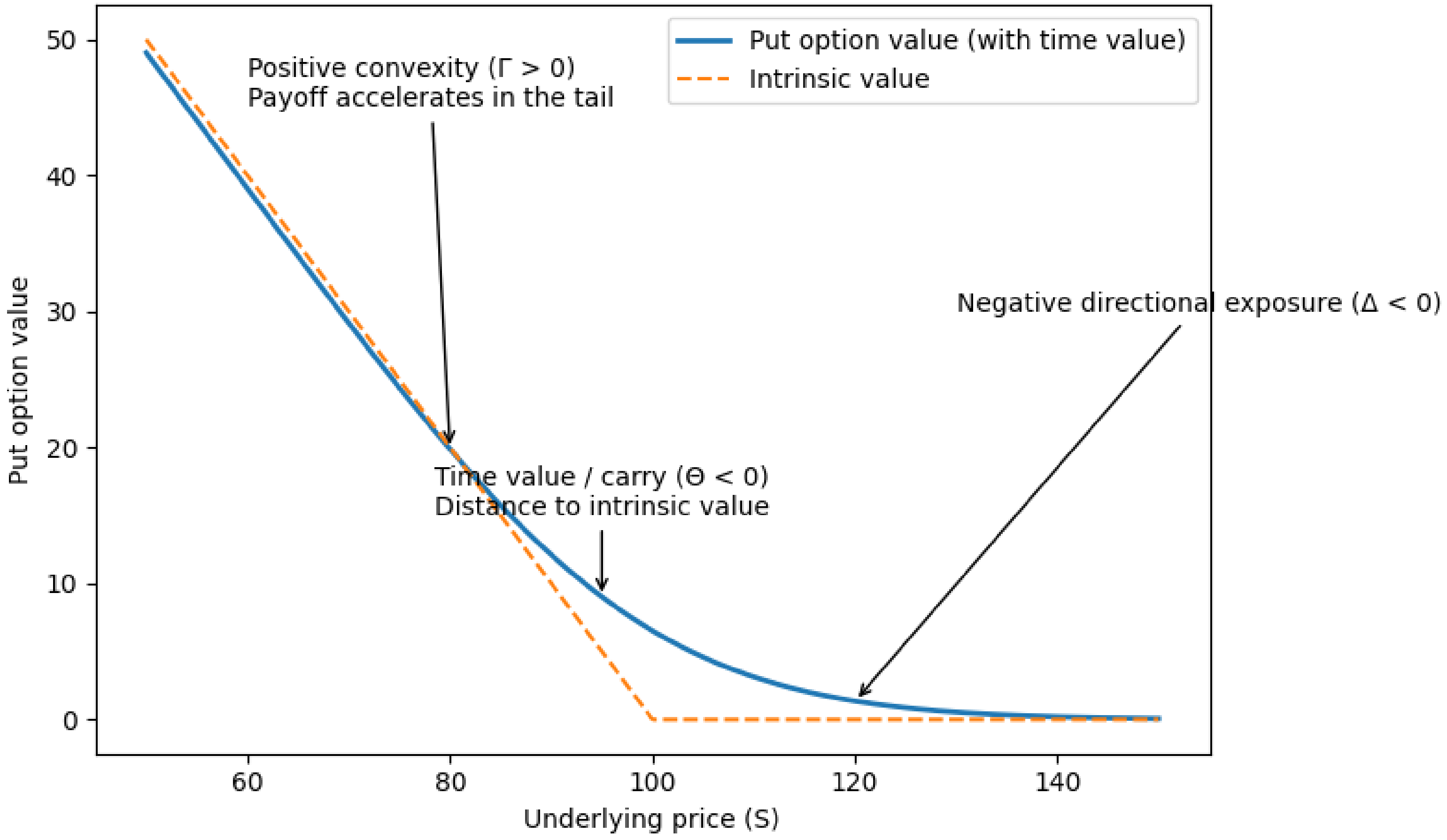
Our approach

- Frame hedge design problem as optimization between carry, convexity & reliability
- Map these concepts to option Greeks and link to P&L attribution
- Derive testable predictions on optimal hedge design validated in backtests

You can’t always get what you want… - the „Greek trilemma”

A long put combines:

- a negative directional exposure to the underlying market,
- a positive exposure to volatility
- a positive convexity
- a negative time value



How much of each ingredient do we want, and how much are we prepared to pay for it?

Greek Trilemma a la Taylor: some theory

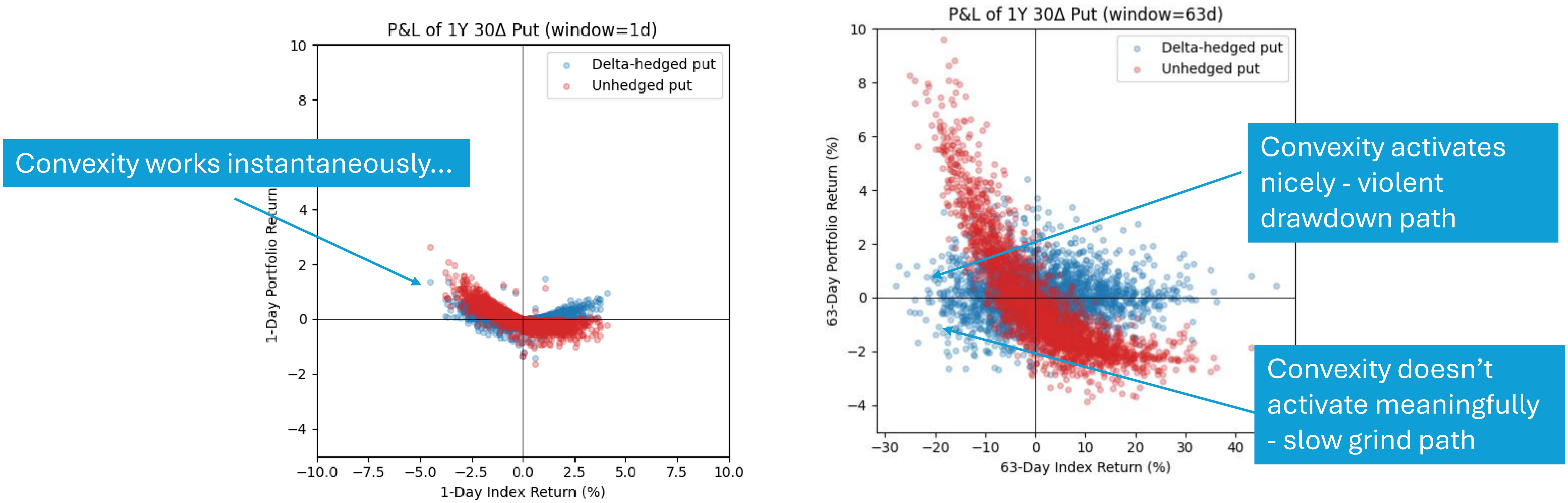
Θ Δ $\Gamma, \nu, \Xi, \Lambda$

$$dP \approx \frac{\partial P}{\partial t}dt + \frac{\partial P}{\partial S}dS + \frac{1}{2}\frac{\partial^2 P}{\partial S^2}dS^2 + \frac{\partial P}{\partial \sigma}d\sigma + \frac{1}{2}\frac{\partial^2 P}{\partial \sigma^2}d\sigma^2 + \frac{\partial^2 P}{\partial S \partial \sigma}dSd\sigma$$

- Δ – **reliability**, path-independent downside protection, but dilutes equity exposure
- {Γ, v, Ξ, Λ} – **convexity** complex, source of incremental P&L in adverse states, but path- & state-dependent (needs sharp spot/vol moves)
- Θ – **carry**, predictable, unconditional financing cost required to maintain a given hedge over time, independent of whether adverse market states occur.

Delta-hedging as a design choice: the good, the bad, the ugly

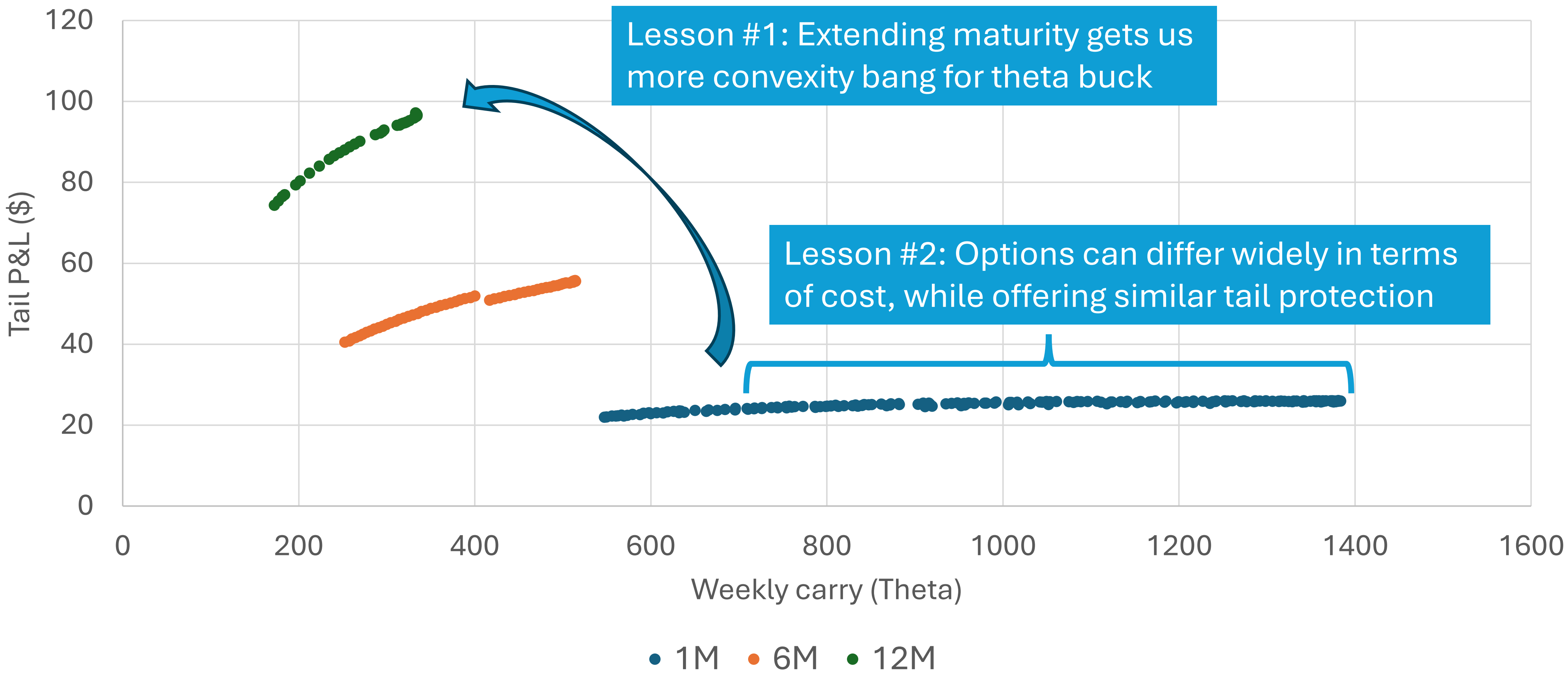
Daily and quarterly realized returns of delta-hedged and unhedged puts



- The **good**: clean convexity without beta bleed
- The **bad**: removes the reliable, always-on protection channel
- The **ugly**: makes the hedge less intuitive, path- & horizon-dependent

Navigating Convexity vs. Carry trade-off: first glance

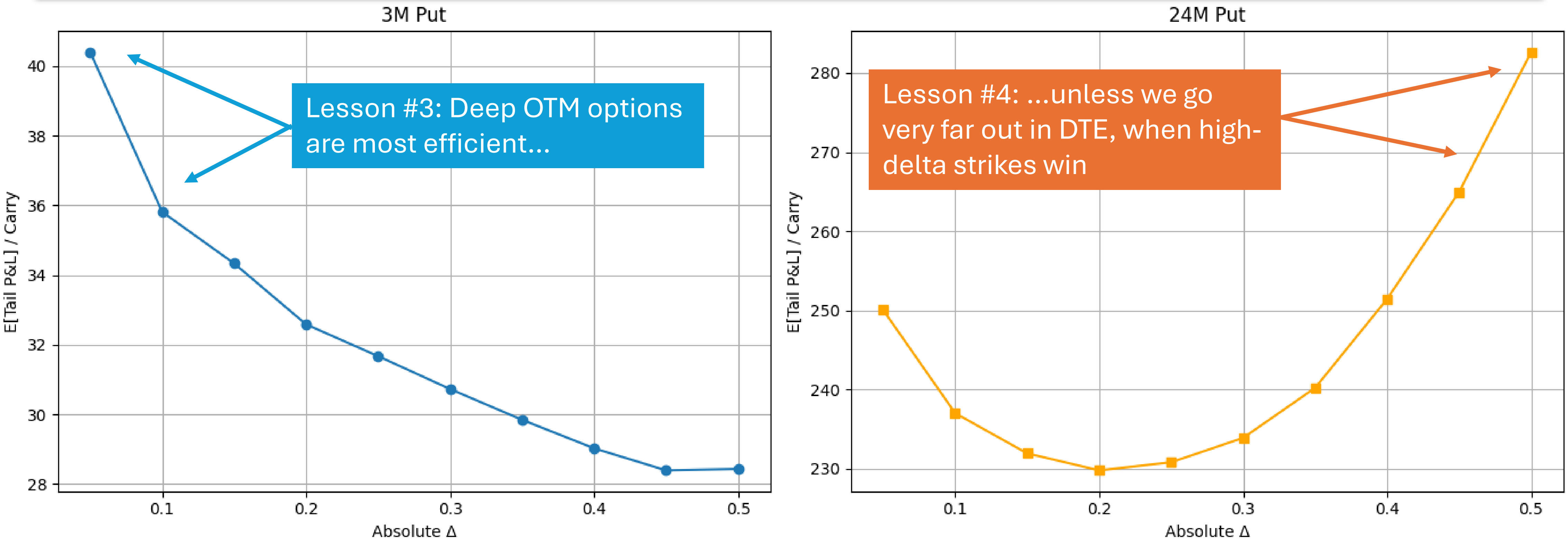
Shock P&L attribution for 1M, 6M & 12M SPX options (~6,000 contracts in total; pricing as of 27 Feb 2025)



Shock assumptions: -10% SPX and +10 vol pts over a week

Convexity vs. Carry trade-off: Monte Carlo experiments

Hedge efficiency as a function of moneyness for a 3M and 24M delta-hedged put

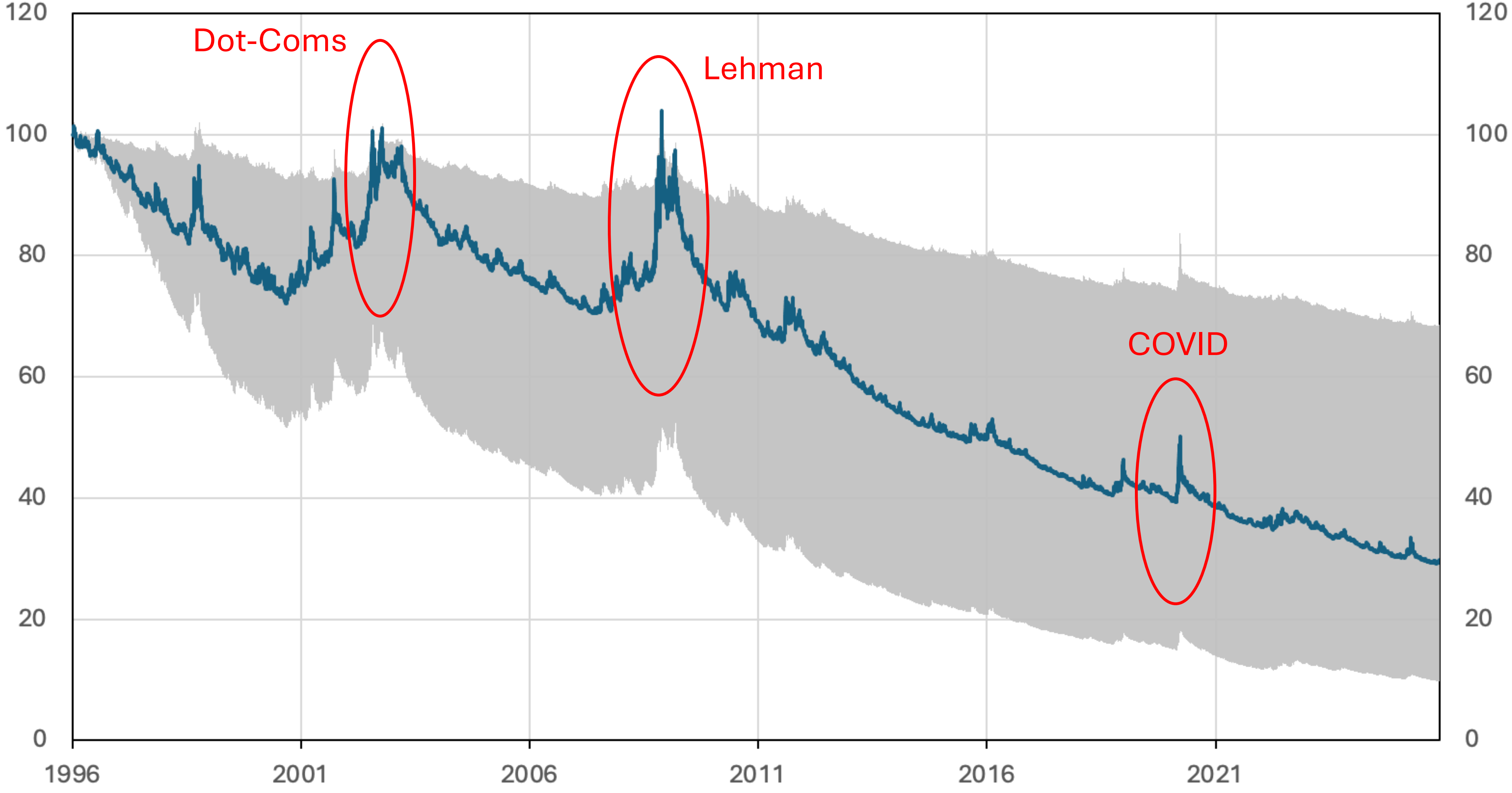


Note: $\mathrm{E}(\mathrm{P\&L}_{\mathrm{tail}})$ is the mean delta-hedged option return conditional on the underlying dropping by more than 2.5%; carry represents the daily theta. To estimate $\mathrm{E}(\mathrm{P\&L}_{\mathrm{tail}})$, we simulate joint shocks to spot and implied volatility using a parametric model: $\frac{dS}{S} \sim N\left(0, \sigma_0\sqrt{dt}\right)$ and $\mathrm{d}\sigma = \beta\left(\frac{dS}{S}\right) + \eta\left(\frac{dS}{S}\right)^2 + \varepsilon$ with $S_0 = 100, \sigma_0 = 20\%$ and $\beta = -1, \eta = 2$.

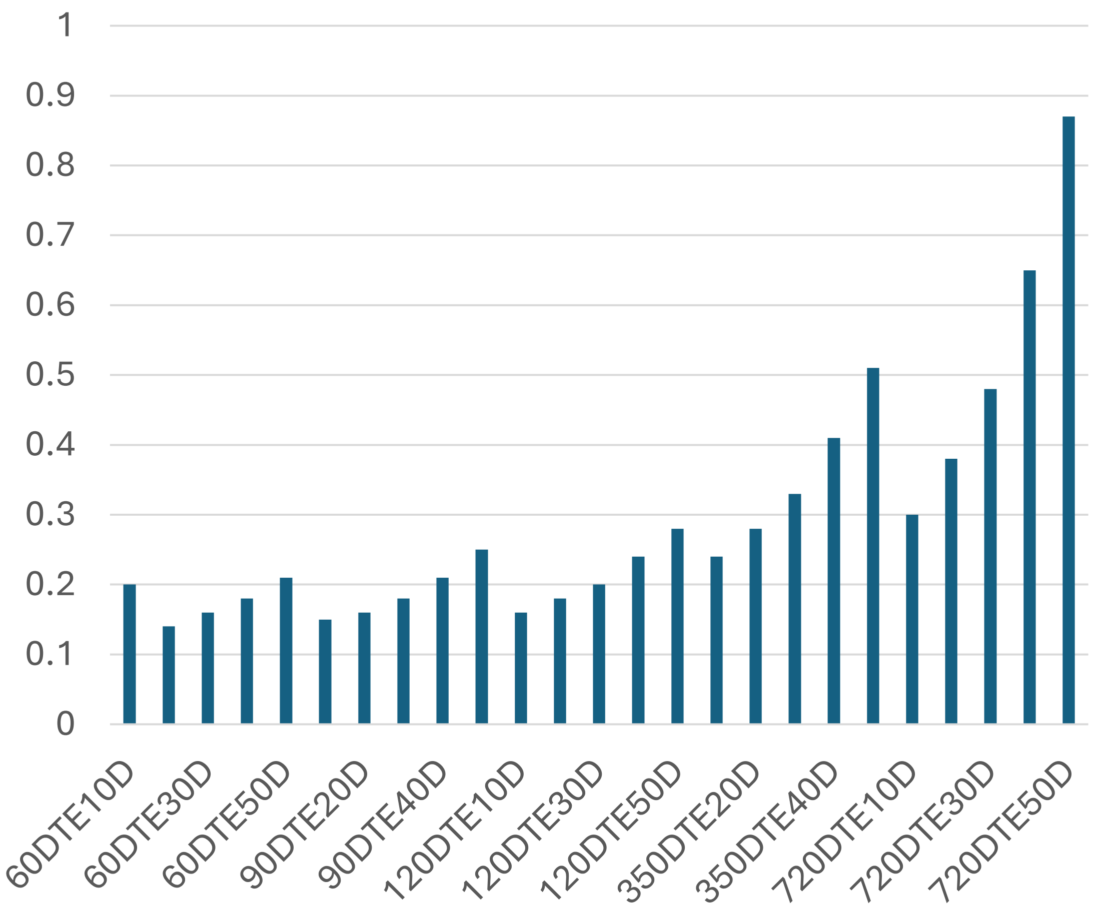
Backtest with KISS in mind

- Universe: European S&P 500 (SPX) put options
  - Maturities: 60, 90, 120, 350, 720 days to expiration
  - Moneyness: 10D, 20D, 30D, 40D, 50D (delta-based strikes)
- Implementation:
  - Monthly/Quarterly rolling into a new option with target maturity and delta
  - Position size scaled to provide 100% notional protection every month
  - Residual cash invested in T-bills
- Sample
  - Daily data, January 1996 – October 2025
  - Option prices, Greeks from OptionMetrics
  - Total-return indices constructed for each strategy

All strategies lose money over time… but mostly deliver when needed



Tail ***payoffs*** differ mostly by delta, hedge ***costs*** – by maturity



Tail event definition

- Non-overlapping horizons:
  - 1D, 1W, 1M, 1Q
- Corresponding SPX drawdown thresholds:
  - −2%, −10%, −15%, −25%
- Tail payoff measured as conditional mean hedge return during these events

Cost (carry) measure

- Ex ante, predictable cost
- Annualized theta of the rolling hedge
- Predictable, worst-case financing rate of convexity at the maturity point repeatedly visited by the strategy

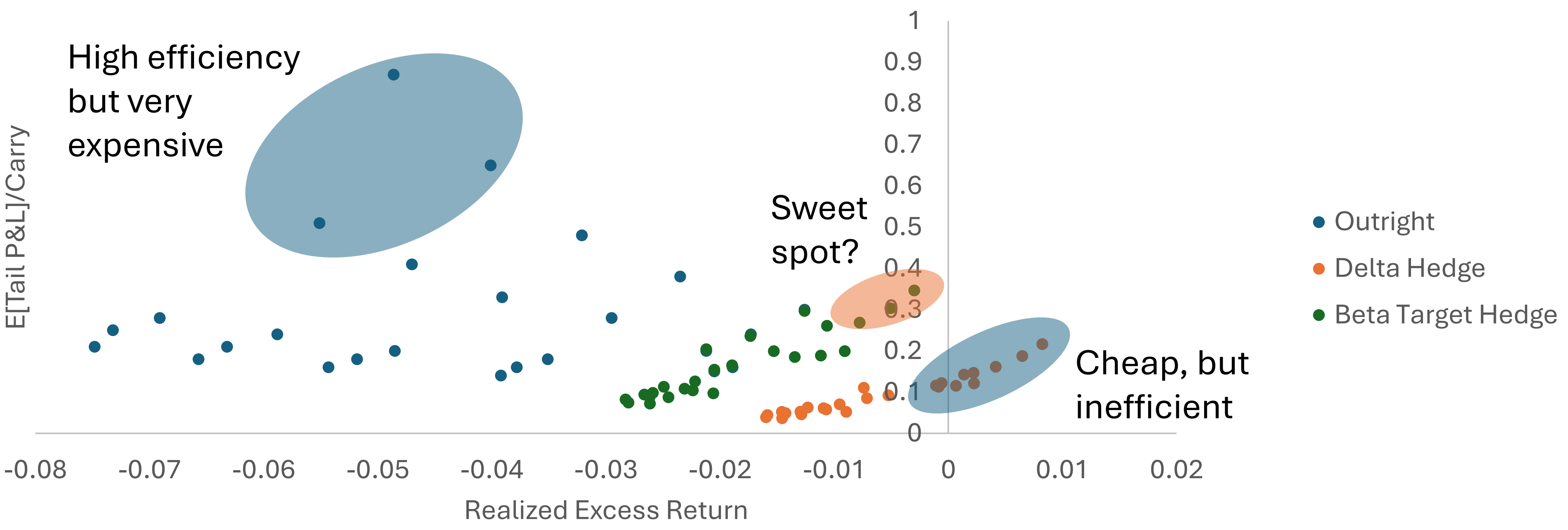
Hedge efficiency=E[Hedge P&L|Tail event]/Carry

Most efficient hedge designs combine higher-delta strikes (stronger conditional tail payoffs) with the longest feasible maturities (lowest carry).

Partial delta hedging as reliability & beta-budgeting

- Put delta → Reliability → equity beta bleed
- Q: "How much of my core equity exposure am I willing to give up, in expectation, to buy tail insurance?"
- Residual portfolio beta ≈ $(1-\alpha) \times \Delta_{PUT}$
- Set α so that all tail hedges impose roughly the same *instantaneous* equity dilution of -0.1 (10D => α=0, 20D=> α=0.5 … 50D=> α=0.8)

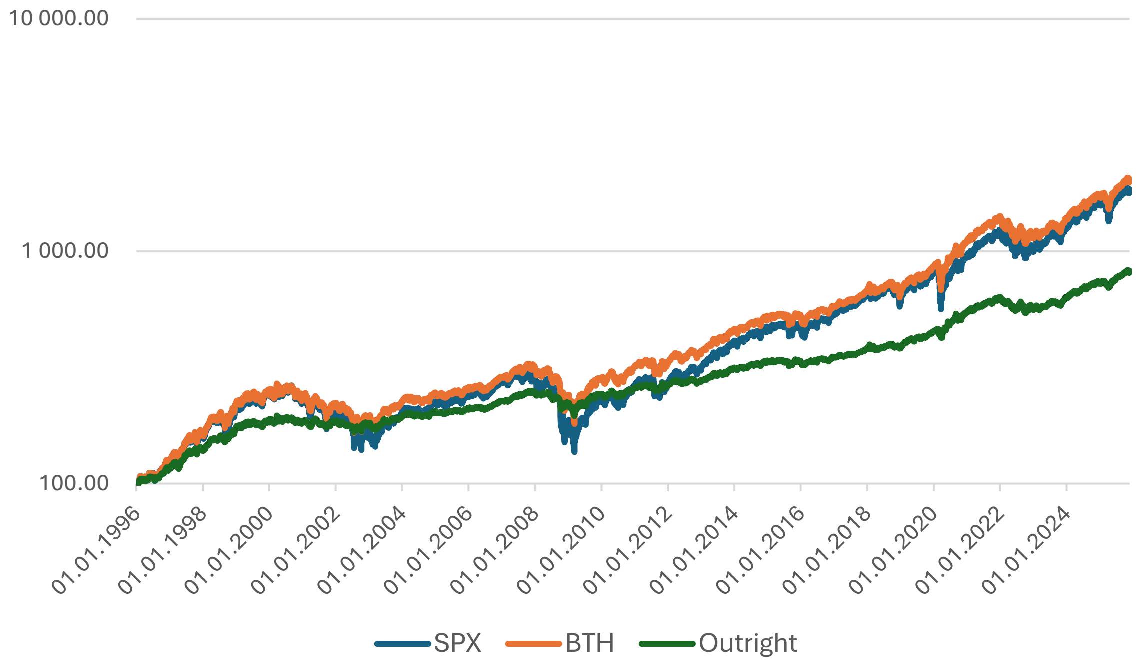
Tail hedge efficiency vs. Realized Excess Return (1996-2025)

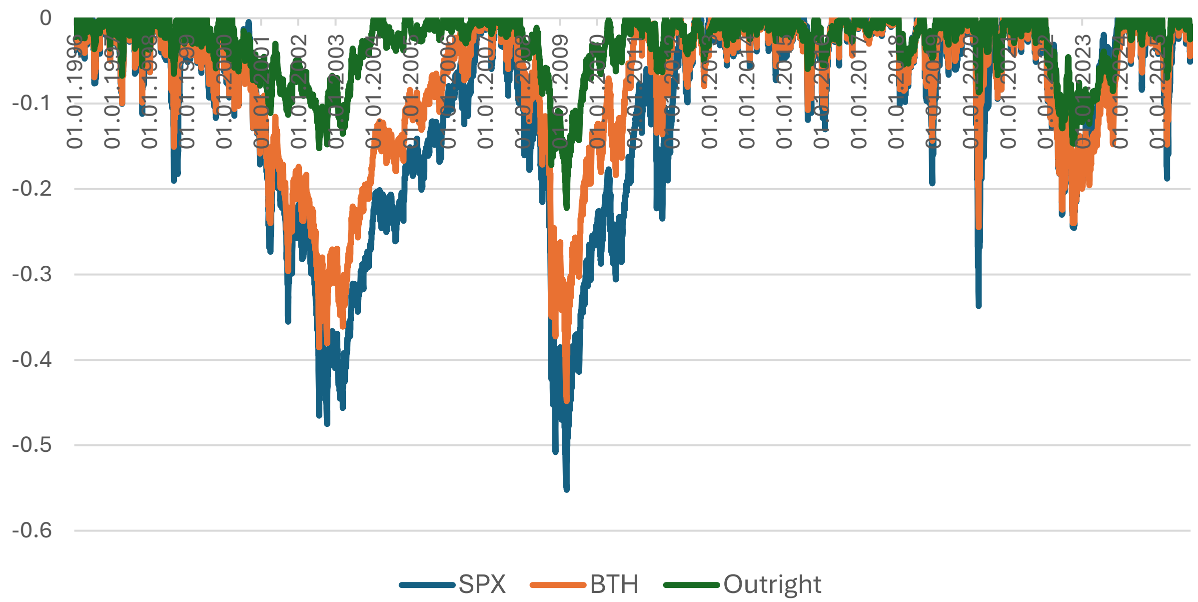


Proof of the pudding is in the eating: tail hedging as an overlay

- Replicate SPX via Futures
- Pledge T-bills as collateral
- Use cash to fund options and reinvest proceeds into T-bills

Strategy (SPX+)	CAGR	Vol	Max DD
SPX	10.24%	19.3%	-55%
720DTE50D – outright	7.30%	8.4%	-22%
720DTE50D – 80% DH	10.61%	16.3%	-45%

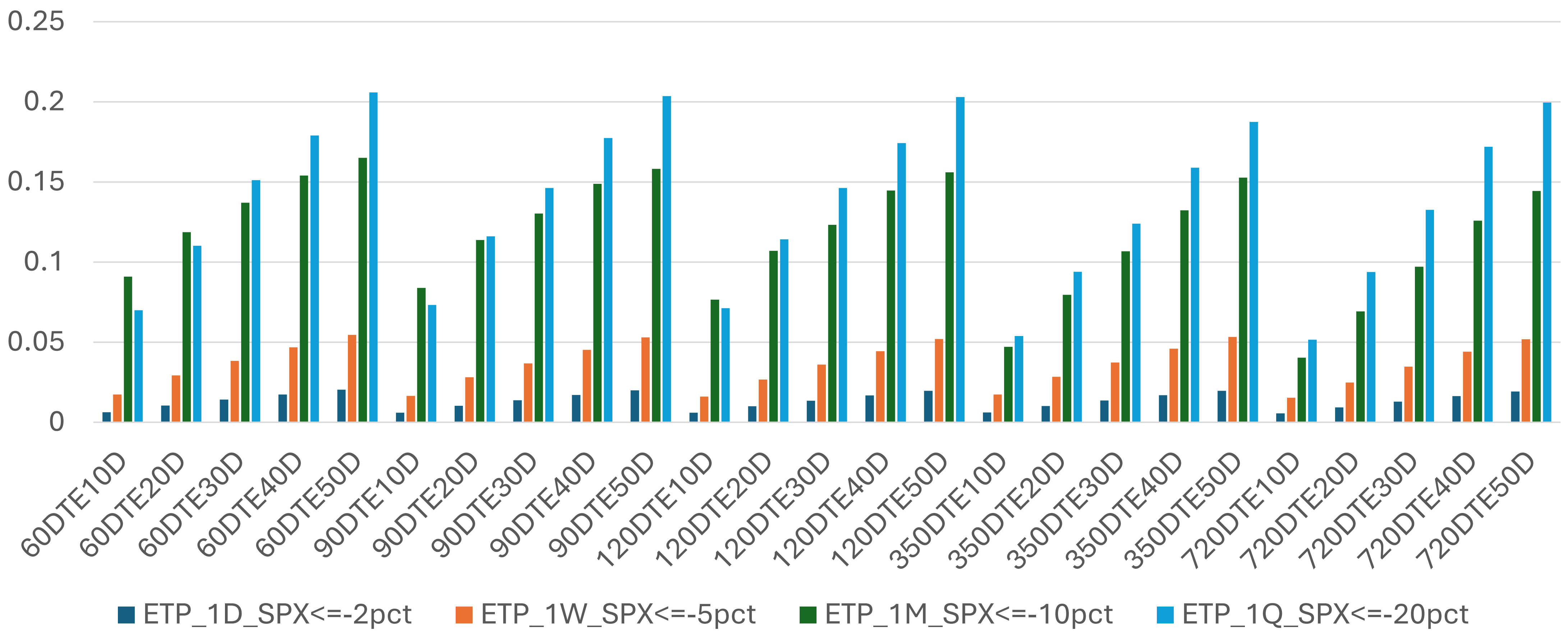




Concluding thoughts

- TRH can benefit portfolios, despite negative drift!
- TRH is about sourcing reliability & convexity as cheaply as possible
- The economic trade-off can be framed as a trilemma between:
    - carry (predictable cost of maintaining protection),
    - convexity (nonlinear payoffs in severe states),
    - reliability (how consistently protection materializes along drawdown paths).
- Key insights:
    - Convexity activation is well preserved over DTE (gamma ↓, but vega ↑), despite clear monotonicity in theta -> **<u>extending DTE best way to improve efficiency</u>**
    - reliability is delivered through delta but dilutes equity exposure and must be rationed through **<u>partial hedging</u>**
- General warning: a position is not a strategy – active beats passive
- Plenty more to do in the space!

Expected tail payoffs across scenarios



Carry per strategy (annualized theta)