The Immaculate Rebalance

February 2025

Introduction

CEO/CIO of Newfound Research LLC

Founded August 2008

Co-Founder of Return Stacked® ETFs

Launched Feb ‘23; $850mm+ in portable alpha strategies

Host of Flirting with Models podcast

7 Seasons of quant interviews

- Rebalance Timing Luck:
  The Difference Between Hired and Fired
  (Journal of Index Investing, 2019)

- Rebalance Timing Luck:
  The (Dumb) Luck of Smart Beta
  (2020)

- The Hidden Cost in Costless Put-Spread Collars: Rebalance Timing Luck
  (Journal of Alternative Investments, 2023)

The Immaculate Rebalance

Fundamental Indexation

Financial Analysts Journal, Vol. 61, No. 2, March/April 2005, pp. 83-99.

35 Pages • Posted: 15 Oct 2004 • Last revised: 30 Dec 2016

Robert D. Arnott
Research Affiliates, LLC

Jason C. Hsu
Research Affiliates; Rayliant Global Advisors; University of California, Los Angeles - Anderson School of Business

Philip Moore
Pacific Investment Consultants, LLC

There are 2 versions of this paper

Date Written: 2005

The Immaculate Rebalance

We rebalance the index only once a year, on the last trading day of the year, for two reasons. First, the financial data available through Compustat are available only on an annual basis in the earliest years of our study. Second, when we try monthly, quarterly, and semi-annual rebalancing, we increase index turnover but find no appreciable return advantage over annual rebalancing.

The Immaculate Rebalance

Their proposed fundamental index is rebalanced once a year at the end of December. The authors note that more frequent rebalancing increases turnover but not average returns. The real-life FTSE/RAFI indexes, which are based on the fundamental indexation concept as outlined in the paper, are also rebalanced once per year. The date chosen in this instance, however, is the third Friday of March.[1]

[1] Quoting from sections 5.1 and 5.2 from the 1.17 version (Nov 2009) of the methodology for the management of FTSE RAFI index series: "*The FTSE RAFI Index Series will be reviewed annually based on data as at the close of business on the last trading day of February, taking into account any additions and deletions planned in the underlying indices. Changes arising from the annual review will be implemented after the close of the index calculation on the third Friday of March each year.*" Source: http://www.ftse.com/Indices/FTSE_RAFI_Index_Series.

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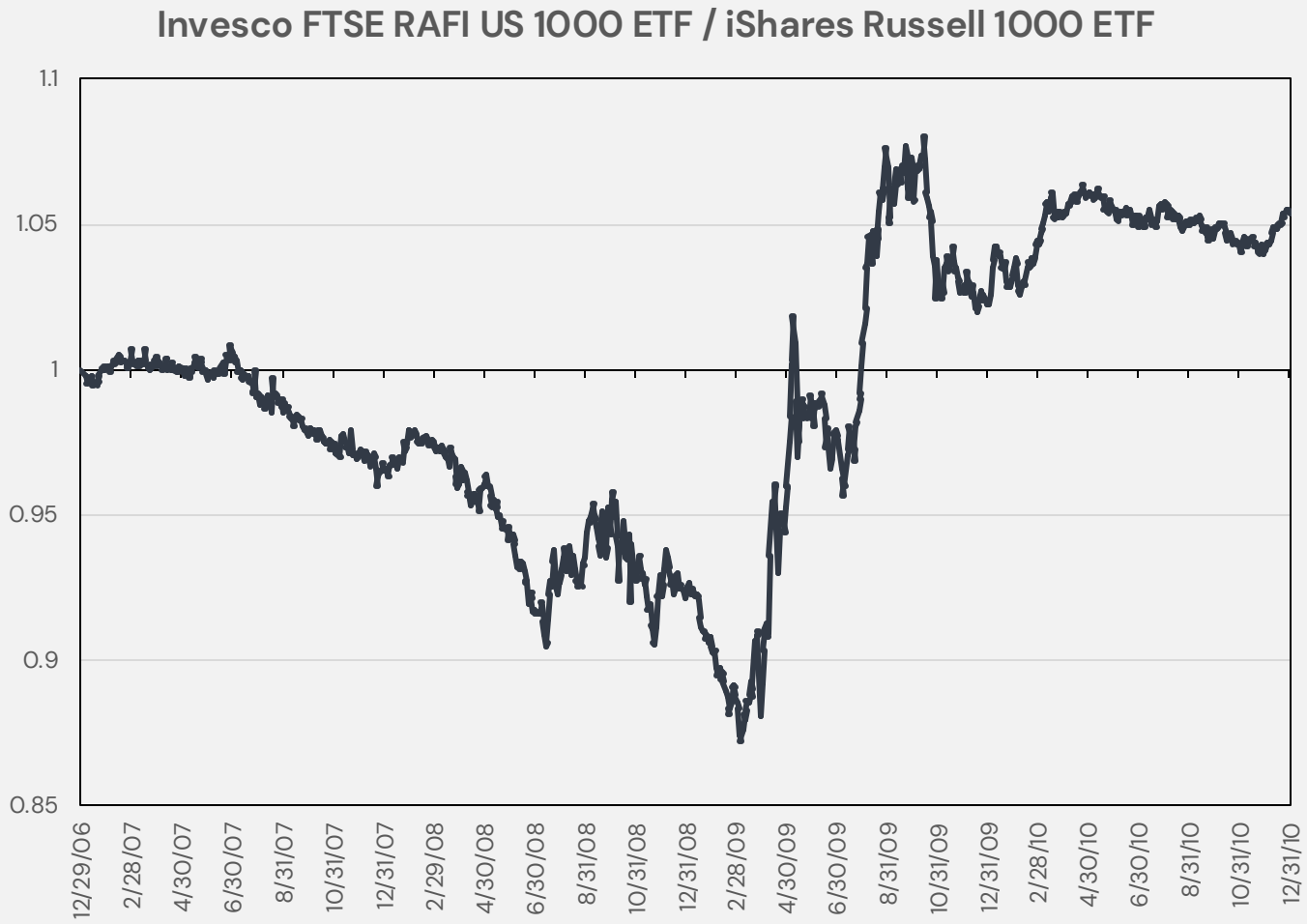

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Fundamental Indexation: Rebalancing Assumptions and Performance

Journal of Index Investing, Vol. 1, No. 2, pp. 82-88, 2010

15 Pages • Posted: 25 Mar 2010 • Last revised: 30 Sep 2010

David Blitz
Robeco Quantitative Investments

Bart van der Grient
Robeco Quantitative Investments

Pim van Vliet
Robeco Quantitative Investments

Date Written: August 3, 2010

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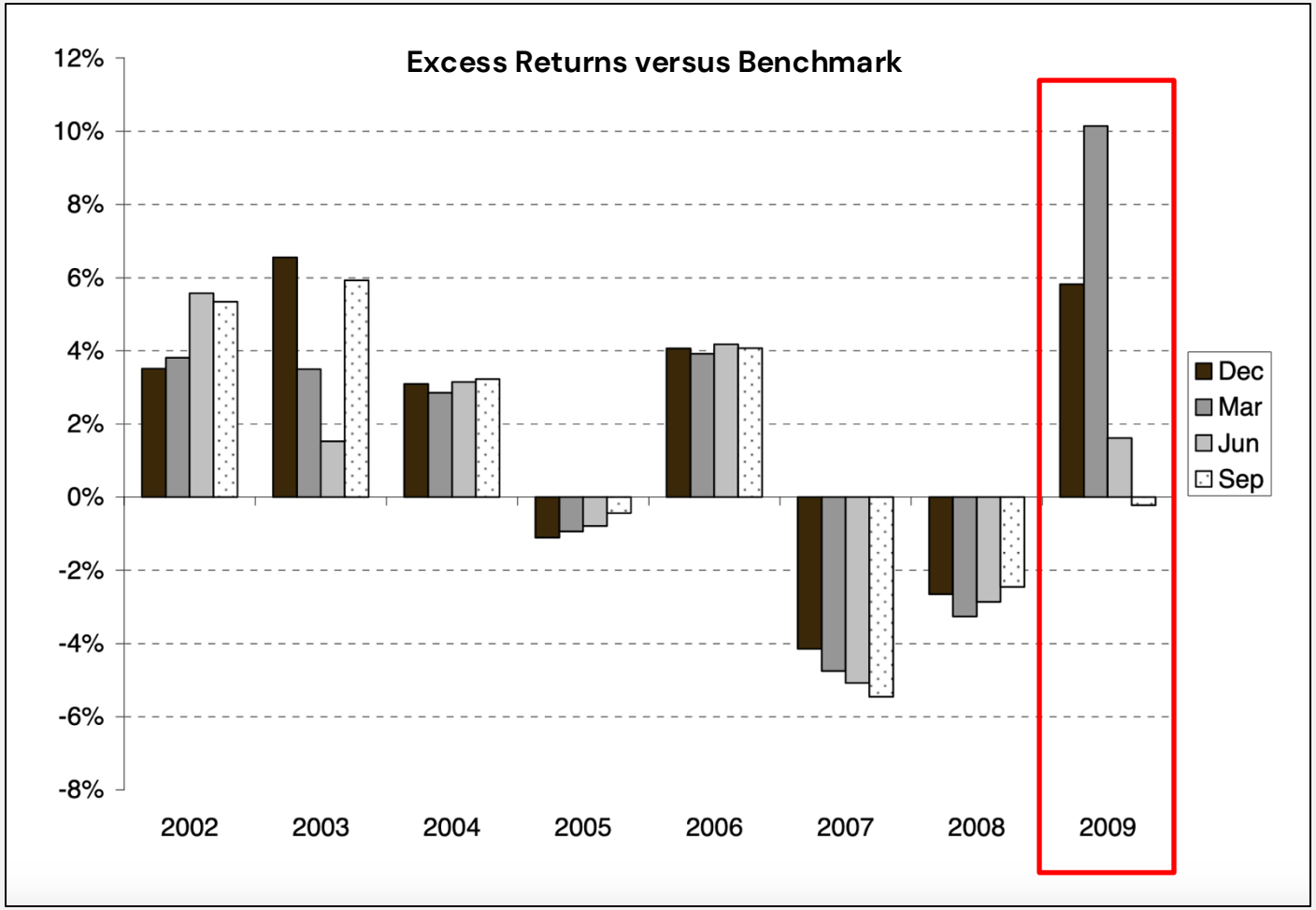
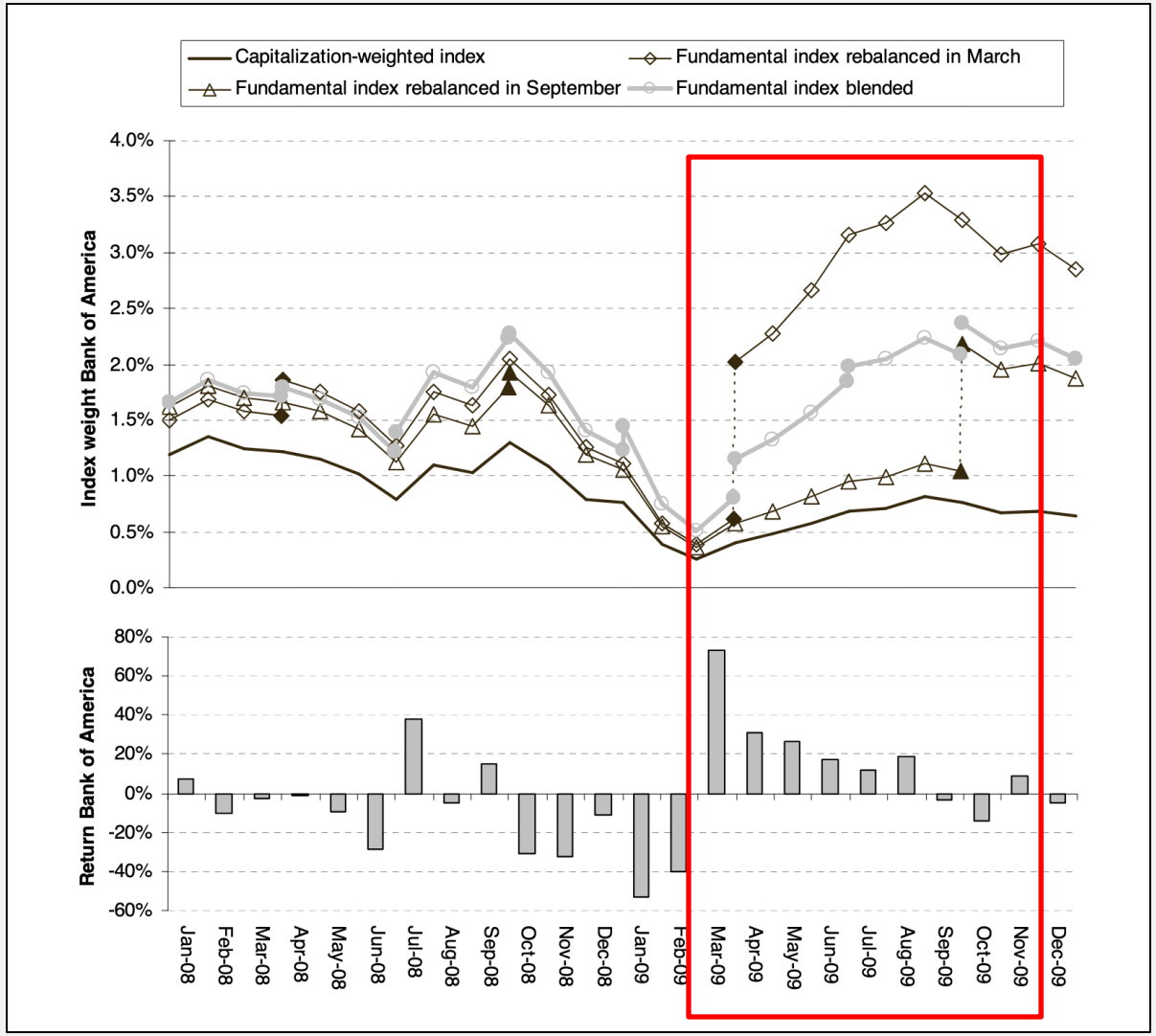


>1000bps

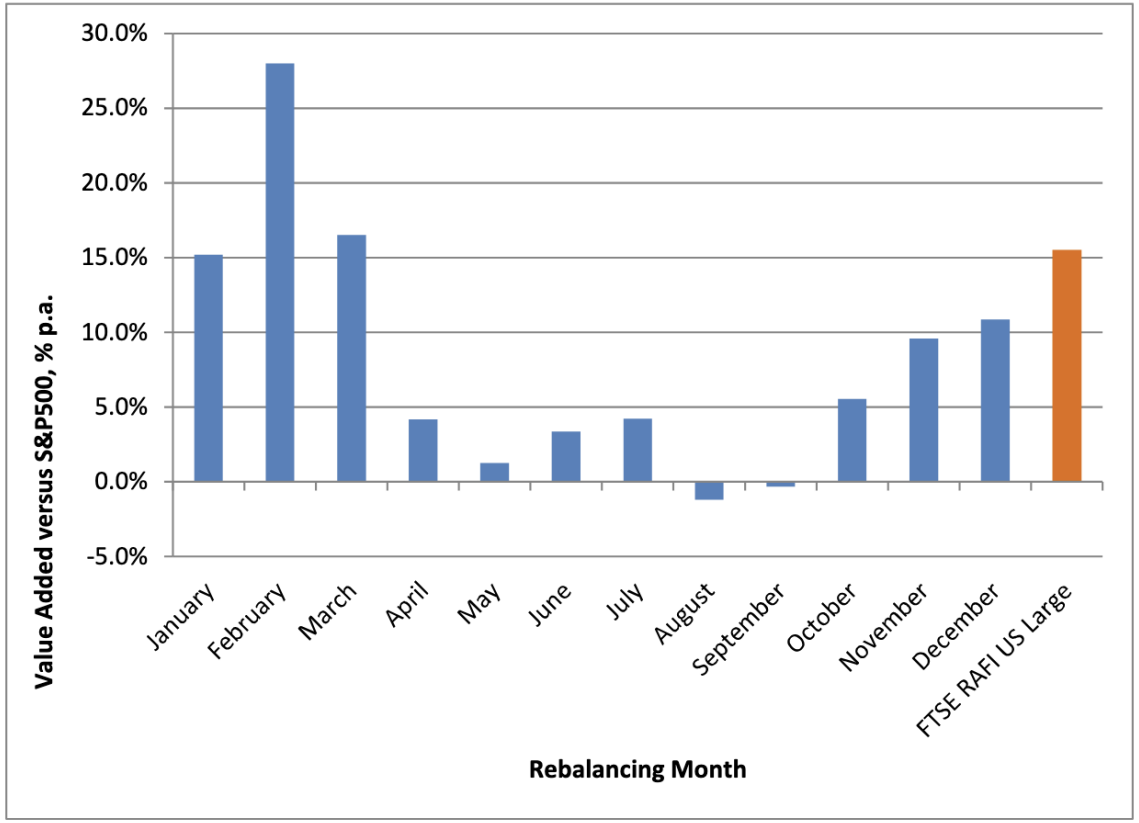
Your Portfolio is Different than My Portfolio



THE IMMACULATE REBALANCE

The Immaculate Rebalance

Figure 1. Value Added of FTSE RAFI US 1000 Index Relative to S&P 500 Index by Rebalance Month, Calendar Year 2009



Source: Research Affiliates, LLC.

Rebalance Timing Luck is…

- Largely arbitrary (but exacerbated by market volatility)
- Non-mean reverting
- A *permanent* impact on portfolio returns
- Can be *very, very* large

Where is the Risk?

- Rebalance timing does *not* impact portfolio volatility

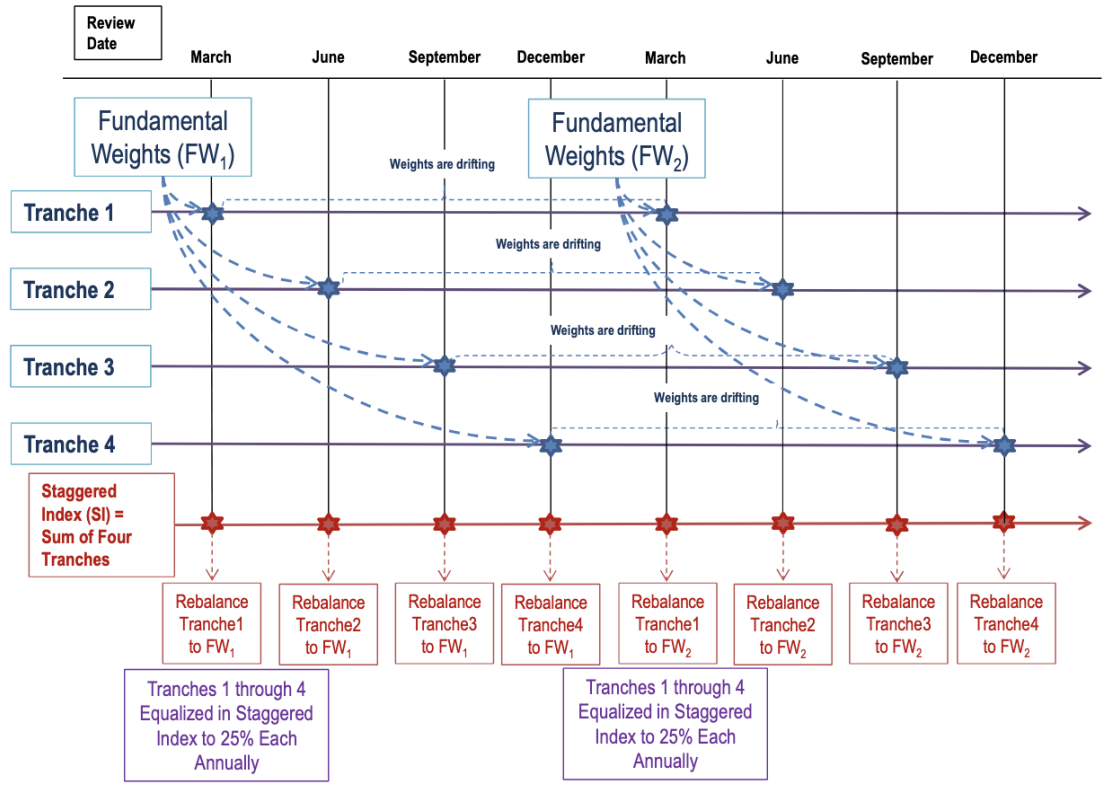
- Rebalance timing *does* impact the distribution of terminal wealth

Why Should We Care?

- It’s an uncompensated risk in your portfolio.
- Your benchmark/competitors can get lucky/unlucky.
- Makes manager due diligence much more difficult.

The Proposed Solution: Staggered Rebalancing

Figure 5. Mechanics of Staggered Rebalancing



Source: Research Affiliates, LLC.

How Do We Measure “Rebalance Timing Luck”?

Empirically:

$$L = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\frac{1}{M-1}\sum_{j=1}^{M}\left(r_{i,j} - \bar{r}_j\right)^2\right)}$$

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Return of discrete rebalance minus staggered rebalance implementation.

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Population variance of discrete implementation minus staggered implementation.

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$$L = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\frac{1}{M-1}\sum_{j=1}^{M}\left(r_{i,j}-\bar{r}_j\right)^2\right)}$$

Debiased mean of the variances

How Do We Measure “Rebalance Timing Luck”?

Theoretical Model:

$$L = \frac{Tf}{2}\sigma$$

How Do We Measure “Rebalance Timing Luck”?

Theoretical Model:

Annualized Turnover Rate

$$L = \frac{Tf}{2}\sigma$$

How Do We Measure “Rebalance Timing Luck”?

Theoretical Model:

Rebalance Frequency

$$L = \frac{T f}{2} \sigma$$

How Do We Measure “Rebalance Timing Luck”?

Theoretical Model:

$$L = \frac{Tf}{2}\sigma$$

“How much continuous turnover do I miss between rebalances?”

How Do We Measure “Rebalance Timing Luck”?

Theoretical Model:

$$L = \frac{Tf}{2}\sigma$$

Tracking error between a single implementation and an infinitely-staggered implementation.

How Do We Measure “Rebalance Timing Luck”?

Theoretical Model:

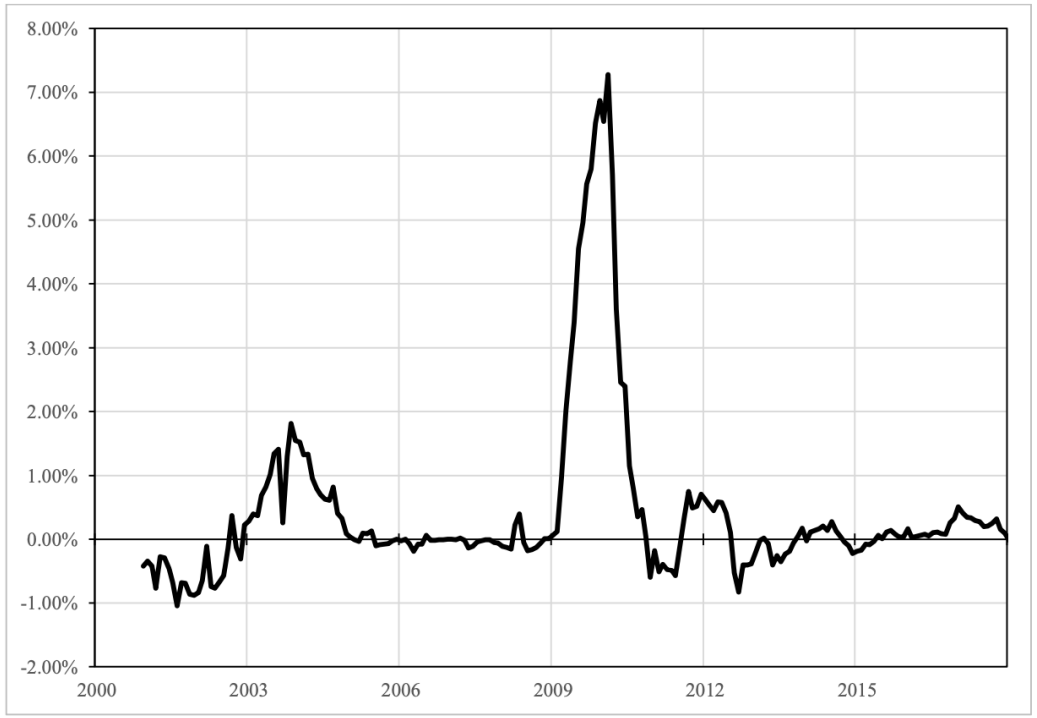
$$L = \frac{Tf}{2}\sigma$$

“How different will my discretely rebalanced portfolio be from a continuously rebalanced portfolio?”

Rebalance Timing Luck in Strategic Portfolios

Exhibit 2

Rolling 1-year total return difference between 60/40 fixed-mix portfolios rebalanced in February and August from 12/1999 to 7/2018.



Source: Global Financial Data. Past performance is not an indicator of future results. Performance is backtested and hypothetical. Performance figures are gross of all fees, including, but not limited to, manager fees, transaction costs, and taxes. Performance assumes the reinvestment of all distributions.

Rebalance Timing Luck in “Smart Beta” Indices

		50	100	150	200	250	300	350	400
Momentum	Annual	7.44%	5.18%	4.01%	3.16%	2.50%	1.96%	1.44%	1.03%
	Semi-Annual	5.92%	4.38%	3.42%	2.59%	2.04%	1.50%	1.13%	0.88%
	Quarterly	4.16%	2.74%	2.15%	1.71%	1.39%	1.10%	0.87%	0.74%
Value	Annual	4.48%	3.17%	2.57%	2.09%	1.73%	1.36%	1.19%	1.03%
	Semi-Annual	3.59%	2.46%	2.01%	1.63%	1.40%	1.29%	1.22%	1.02%
	Quarterly	2.78%	1.79%	1.60%	1.34%	1.15%	0.96%	0.88%	0.74%
Quality	Annual	3.35%	2.40%	1.97%	1.57%	1.41%	1.31%	1.31%	1.37%
	Semi-Annual	2.81%	2.02%	1.73%	1.43%	1.34%	1.22%	1.16%	1.23%
	Quarterly	1.97%	1.31%	1.18%	0.99%	0.83%	0.77%	0.82%	0.81%
Low Volatility	Annual	2.15%	1.42%	1.22%	1.05%	0.99%	0.92%	0.87%	0.83%
	Semi-Annual	1.58%	1.13%	0.90%	0.75%	0.71%	0.66%	0.67%	0.66%
	Quarterly	1.08%	0.79%	0.59%	0.56%	0.51%	0.42%	0.41%	0.47%

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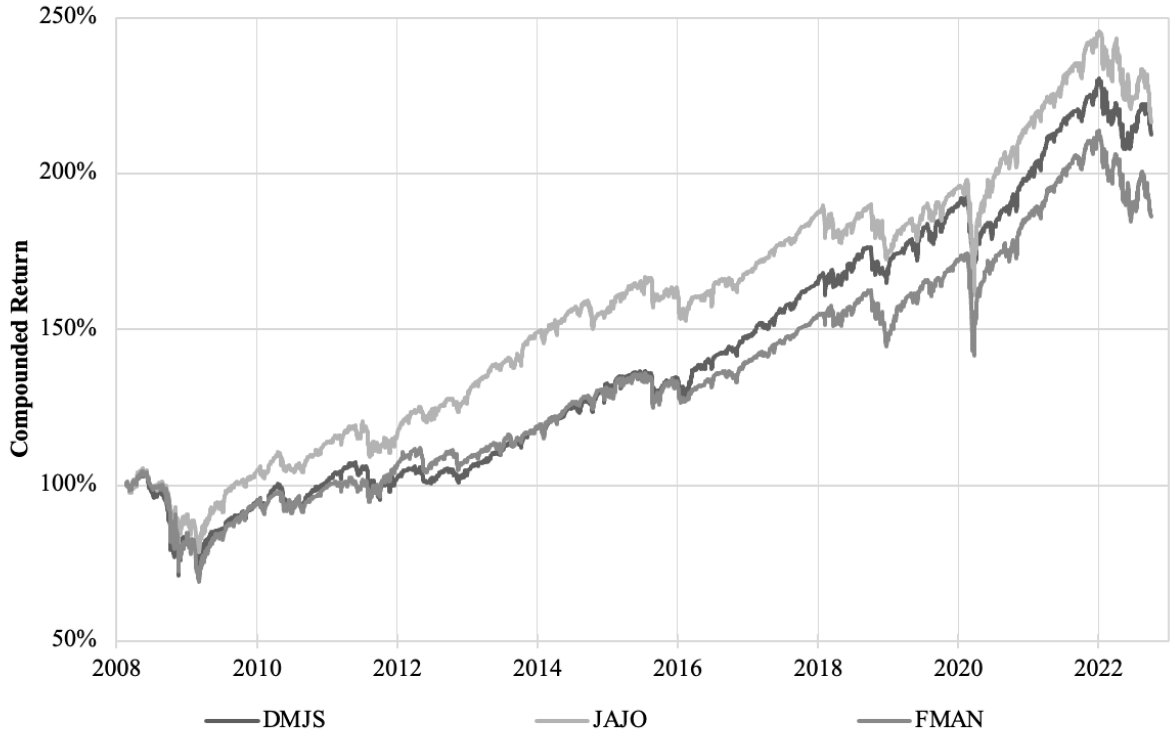
Rebalance Timing Luck in Option Strategies

Exhibit 1: Compound Excess Returns

Compound excess returns for the three different rebalance schedule implementations of the self-financing put-spread collar strategy.

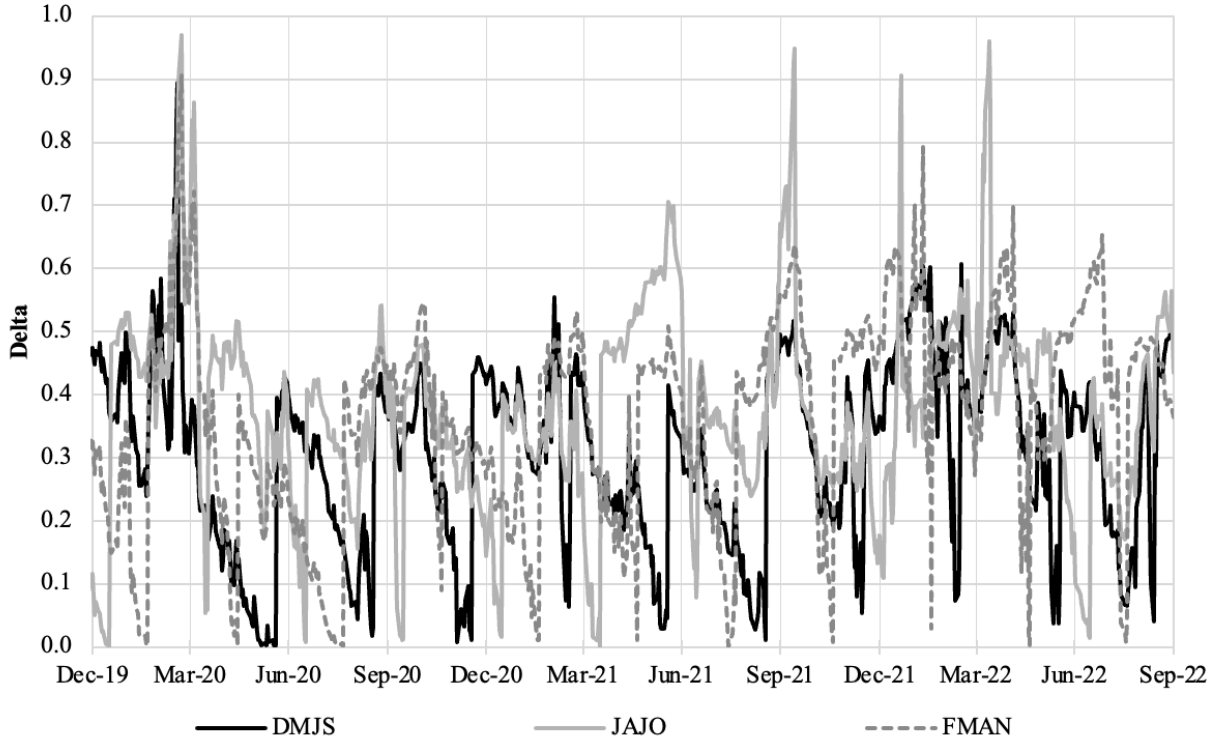


Source: Newfound Research, NDVR, Reuters and iVolatility. Past performance is not indicative of future returns. Performance is backtested and hypothetical. Performance figures are gross of all fees, including, but not limited to, manager fees, transaction costs, and taxes. Performance assumes the reinvestment of all distributions.

Rebalance Timing Luck in Option Strategies

Exhibit 8: Strategy Net Delta Over Time

Net delta profiles of each strategy implementation plotted over time.



Source: Newfound Research, NDVR, and iVolatility.

But How Different Can Two Implementations Be?

If we run an identical process...

And rebalance on an identical frequency...

But rebalance on a different schedule...

Then, in expectation, the tracking error of our returns will be:

$$\sqrt{2}L$$

Who Cares?

Our paper estimated L = 292bp. $\sqrt{2}L$ = 412bp.

412bp x $\sqrt{8/12}$ = 336bp. So, −14.8% vs −7.7% is a ~2.1σ move.

Does Staggering Solve the Problem?

Assume infinite tranches with identical methodologies

$$r_i = \bar{r} + \varepsilon_i$$

In allocating across tranches, our goal is to minimize the contribution of ε_i to overall portfolio variance.

Does Staggering Solve the Problem?

$$\vec{w}^T \Sigma \vec{w} = \sum_{i=1}^{4}\sum_{j=1}^{4} w_i w_j cov(r_i, r_j) \quad (2)$$

$$= \sum_{i=1}^{4}\sum_{j=1}^{4} w_i w_j cov(X + \varepsilon_i, X + \varepsilon_j) \quad (3)$$

$$= \sum_{i=1}^{4}\sum_{j=1}^{4} w_i w_j \sigma^2 + w_i w_j cov(\varepsilon_i, \varepsilon_j) \quad (4)$$

$$= \sum_{i=1}^{4}\sum_{j=1}^{4} w_i w_j \sigma^2 + \sum_{i=1}^{4}\sum_{j=1}^{4} w_i w_j cov(\varepsilon_i, \varepsilon_j) \quad (5)$$

Minimizing this element is equivalent to finding the minimum volatility portfolio of long/short portfolios: $\varepsilon_i = r_i - \bar{r}$

Does Staggering Solve the Problem?

Good news! Minimum variance has a closed form!

$$\vec{w} = \frac{\Sigma^{-1}\vec{1}}{\vec{1}^{T}\Sigma^{-1}\vec{1}}$$

Bad news: what in the world is Σ?

Does Staggering Solve the Problem?

We're going to handwave the math.

$$\Sigma = \sigma_L^2 \begin{bmatrix} 1 & a & b & a \\ a & 1 & a & b \\ b & a & 1 & a \\ a & b & a & 1 \end{bmatrix}$$

The matrix is both symmetric and circulant...

Does Staggering Solve the Problem?

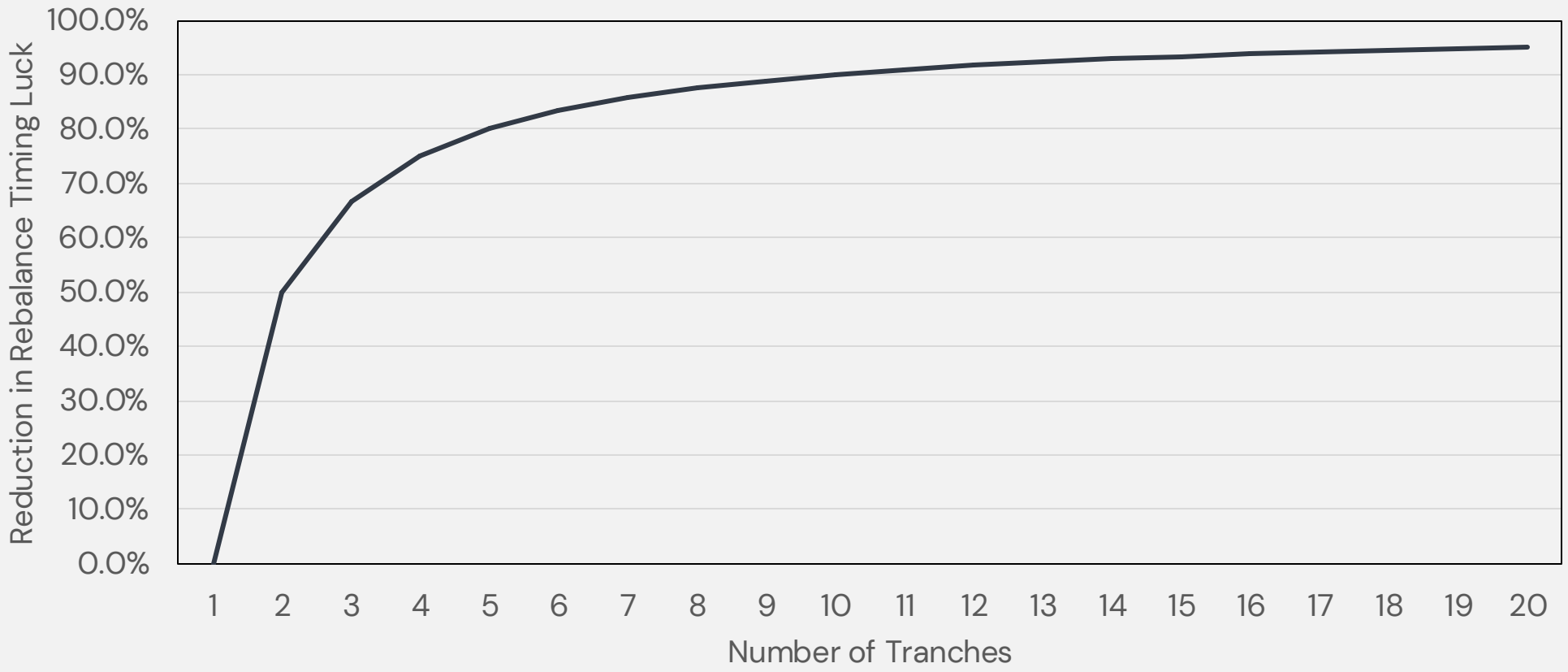
Which means its *inverse* is also symmetric and circulant, which guarantees:

$$\Sigma^{-1}\vec{1} = k\vec{1}$$

If we simplify our original closed-form minimum variance equation, we get 1/N (i.e. equal weight).

How Many Tranches Do We Need?

Timing luck reduces at a rate of 1/N



Frequency and *Schedule* are Different Concepts

> We rebalance the index only once a year, on the last trading day of the year, for two reasons. First, the financial data available through Compustat are available only on an annual basis in the earliest years of our study. Second, when we try monthly, quarterly, and semi-annual rebalancing, we increase index turnover but find no appreciable return advantage over annual rebalancing.

Frequency of rebalancing should be defined by the forecast period of your alpha and t-costs.

When is an arbitrary decision.

What About Seasonality?

- It’s possible for some rebalance schedules to provide an edge
- Usually it’s a very, very low signal compared to rebalance timing luck noise
- Can simply re-run the staggered portfolio optimization, but instead of minimum variance find the maximum information ratio

The Key Takeaways

- *When* you rebalance is an uncompensated risk in your portfolio
- Rebalance timing luck affects every type of strategy
- Unchecked, the magnitude of impact can be *thousands* of basis points in a single year
- Rebalancing “a little bit, frequently” goes a long way in curbing this risk

Thank You

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