Robust optimization in algorithmic investment strategies

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Quantitative Finance Research Group
Faculty of Economic Sciences at University of Warsaw

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Agenda I

- Motivation, hypotheses and research questions
- Literature review
- Data description
- Methodology
  - Performance metrics
  - Walk forward optimization
  - Individual strategies
    - Moving average crossover
    - Sell in May and go away
    - ARIMA
    - Macro-economic factor
    - Summary of crucial assumptions
  - Portfolio of strategies - Signal combination

Agenda II

- Empirical results
    - Individual strategies
    - Portfolio of algorithmic strategies - Signal combination
- Sensitivity analysis
    - Individual strategies
    - New underlying instrument - Nasdaq Composite
    - Ensemble model for S&P500 and Nasdaq
- Research hypotheses verification
- Conclusions
- Research extensions

Motivation

The main aims are:

- the verification of Long/Short algorithmic investment strategies on equity index market,
- the analysis of the set of uncorrelated investment strategies based on different logics such as trend-following, contrarian approach, statistical methods, and macro-economic news,
- to design the proper architecture of algorithmic investment strategies following a personalized Walk-Forward optimization, in which:
  - the model seeks to choose the most robust combination of parameters rather than the best one, in terms of risk-adjusted returns.
  - the model selects intra-period optimal parameters instead of those which were the best in the last in-sample period
- to test the robustness of presented strategies with regards to parameters set at the beginning,

because **none of the previous works covered this topic extensively enough to increase the probability of at least repeating the in-sample results in the out-of-sample period**

Hypotheses and research questions

First Hypothesis:

the market price reflects all public information, and therefore, it is impossible to obtain higher risk-adjusted returns than the market itself

Second Hypothesis:

whether it is possible to obtain better risk-adjusted returns by combining signals from several investment strategies, than from each of them individually.

Third Hypothesis:

whether it is possible to obtain better risk-adjusted returns by combining signals from more than one basis instrument instead of only one

Fourth Hypothesis:

whether methodological advancement (robust optimization criterion and intraperiod selection of optimal parameters) can results in better risk-adjusted returns in out-of-sample period)

Fifth Hypothesis:

whether the final results are robust to initial selction of parameters

Software, Libraries, Hardware

- macOS Big Sur 11.6
- Python 3.7.9
- data source: yfinance 0.1.63
- pandas 1.1.1, numpy 1.19.1
- ARIMA model: pmdarima 1.8.0
- Visualization: matplotlib 3.3.1, seaborn 0.10.1
- Processor: 2GHz Quad-Core Inter Core i5
- RAM: 16GB 3733 MHz

Literature review

Literature review

- **Park et al. (2007)** showed that among a total of 95 modern studies, 56 studies found positive results regarding technical trading strategies, 20 studies obtain negative results, and 19 studies indicate mixed results.
- **Bailey et al. (2013)** referred to the proper and robust optimization process and underlined that the optimization procedure has several pitfalls related to the risk of overfitting, which occurs when a model targets particular data periods rather than a general structure.
- **Devi et. al. (2013),** forecasted Nifty Midcap50 companies with ARIMA model with different parameters. The Box-Jenkins methodology is used to identify the model, and AIC BIC test criteria is applied against the data represented in the past to select the best model. This work generates investment decisions based on the minimum error percentage.
- **Gayed (2016)** found that being exposed to equities with leverage in an uptrend and rotating into risk-free Treasury bills in a downtrend leads to the significant outperformance over time.

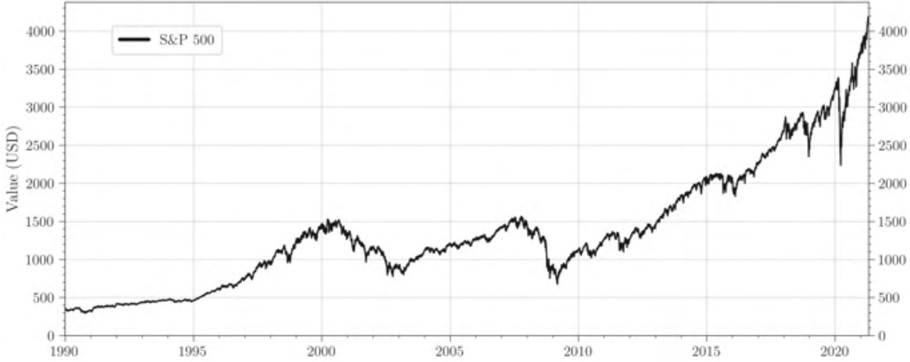
Literature review

- **Ślepaczuk et. al. (2018)** prepared more complex work that proved that is possible to beat the market consecutively by building a portfolio of various investment strategies on various types of asset classes.
- **Korzeń and Ślepaczuk (2019)** analyzed several macroeconomic factors to filter momentum investment strategies on the S&P 500 Index over the last 10 years. The conclusion was that the results of the momentum strategy with a macroeconomic filter were worse than the momentum strategy alone. The benchmark outperformed both strategies in terms of risk-adjusted returns.
- **Boehmer et al. (2020)** studied the impact of algorithmic trading on market quality from 2001 to 2011 in 42 equity markets and concluded that AT improves liquidity and informational efficiency, but increases short-term volatility.
- **Kijewski and Ślepaczuk (2020)** proposed a method which combines signals from several strategies to diversify the risk of wrong predictions by a single strategy. They showed that it is possible to double the compounded returns of S&P 500 index on the same level of risk.

Data description

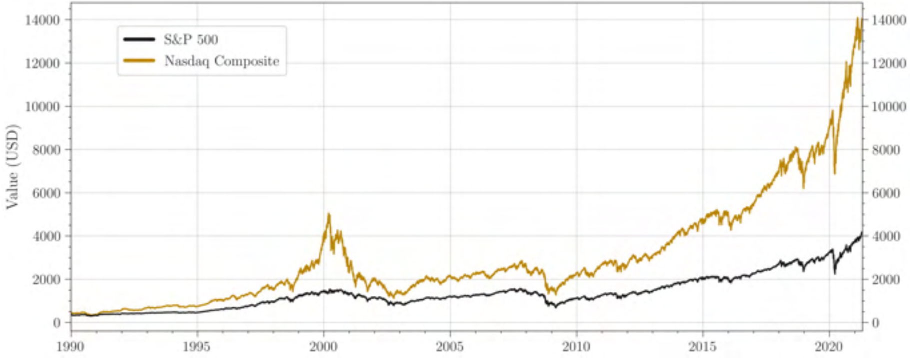
Data description. Standard & Poor 500 index (S&P 500)

- Source: https://finance.yahoo.com/
- Data period: Close prices with daily frequency from 1980-01-01 to 2021-04-23
- Out-of-Sample period: 1990-01-01 to 2021-04-23



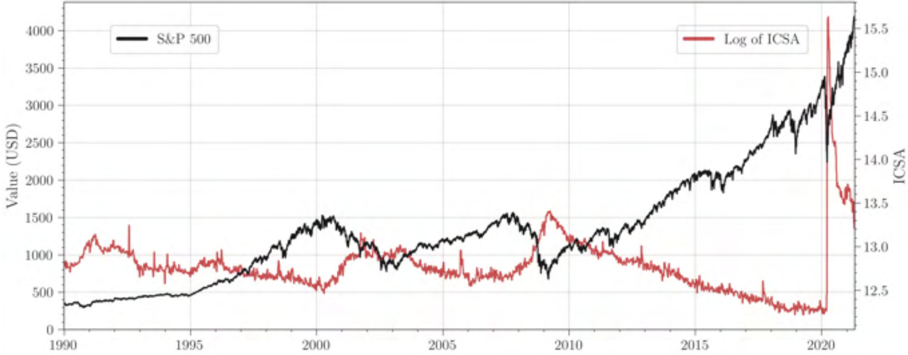
Data description. Nasdaq Composite index (Nasdaq)

- Source: https://finance.yahoo.com/
- Data period: Close prices with daily frequency from 1980-01-01 to 2021-04-23
- Out-of-Sample period: 1990-01-01 to 2021-04-23



Data description. Initial Claims Seasonally Adjusted

- The Initial Claims Seasonally Adjusted (ICSA) requests a determination of basic eligibility for the Unemployment Insurance program. Released every Thursday with the official data for the last Friday.
- Source: https://fred.stlouisfed.org
- Data period: 1980-01-01 to 2021-04-23



Assumptions

Methodology. Assumptions

- It is possible to buy and sell any fraction of S&P 500 futures contracts, including several decimals.
- A commission of 0.02 % is added to every transaction.
- Strategies uses daily close prices in order to generate the forecast for the next day. In practice it requires that the investor will take the price a few minutes before the close of the trading day and change the position accordingly.

Performance Metrics

Methodology. Performance Metrics

- Annualized return compounded (ARC):

$$ARC = \prod_{i=1}^{n} (r_i + 1)^{252/n} - 1 \tag{1}$$

- Annualized standard deviation (aSD):

$$aSD = \sqrt{252} * \frac{1}{n-1} * \sum_{i=1}^{n} (r_i - \overline{r})^2 \tag{2}$$

- Information ratio* (IR*):

$$IR^* = \frac{ARC}{aSD} \tag{3}$$

Methodology. Performance Metrics

- Maximum drawdown (MD):

$$MD = sup_{x,y \ \epsilon \ \{[t_1,t_2]^2 \ : \ x \leq y\}} \frac{P_x - P_y}{P_x} \tag{4}$$

- Average maximum drawdown (AMD):

$$AMD = \frac{\sum_{i=1}^{n} MD_i^{yearly}}{n} \tag{5}$$

- Maximum Loss Duration (MLD): longest time needed to surpass a maximum value of the strategy returns. Measured in years.

- Information ratio** (IR**):

$$IR^{**} = \frac{ARC * ARC * sign(ARC)}{aSD * MD} \tag{6}$$

Methodology. Performance Metrics

- All risk:

$$All\ Risk = aSD * MD * AMD * MLD \tag{7}$$

- Annualized return compounded / Maximum drawdown (ARC MD):

$$ARC\ MD = \frac{ARC}{MD} \tag{8}$$

- Annualized return compounded / Average maximum drawdown (ARC AMD):

$$ARC\ AMD = \frac{ARC}{AMD} \tag{9}$$

- Num. trades: Sum of all changes in position on S&P 500 index.
- No signal: Number of days with a neutral position on S&P 500 index.

Walk forward optimization

Methodology. Walk forward optimization

Advantages over traditional optimization method (one IS and one OOS period):

- Allows the strategy to adapt itself to different market regimes, allowing each strategy to choose different parameters for different periods.
- It produces a longer overall OOS, as at the end of the optimization, all OOS windows are combined creating one large OOS sample.

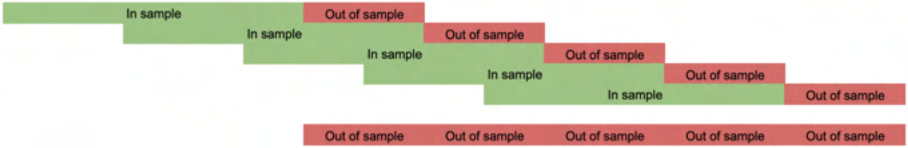
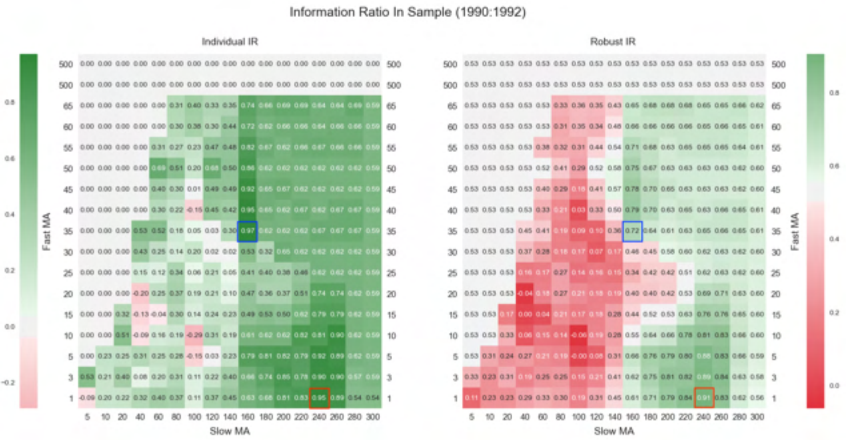


Figure 1: Walk-Forward optimization

Methodology. Robust optimization. Robust IR

The decision about the best performing combination of parameters on each IS window is based on a *robust IR*.

$$Robust\ IR = \frac{IR + average(IR_{neighbors})}{2} \tag{10}$$



Methodology. Robust optimization. Intra-period optimal parameters

Furthermore, this work studies an algorithm that aims to choose the combination that has a more robust performance over time. Such algorithm assigns weights to each parameter combination proportionally to the sum of all *robust_IR* from each IS period. The logic of the algorithm is as follows:

1. Calculate *IR* from all combinations of parameters.
2. Compute the *robust IR*.
3. Calculate weights assigned to each combination of parameters.

$$weight_x = \begin{cases} \min\left(\frac{Robust\ IR_x}{\sum_{i=1}^{n} \max(0, Robust\ IR_i)}, \frac{10}{n}\right) & \text{if } Robust\ IR_x > 0 \\ 0 & \text{if } Robust\ IR_x \leq 0 \end{cases} \tag{11}$$

where:

$$Robust\ IR_x - is\ the\ robust\ IR\ of\ each\ combination\ with\ positive\ IR$$

$$n - is\ the\ total\ number\ of\ tested\ combinations$$

4. Add weights from the most recent *m* IS windows.
5. Choose the combination of parameters with the highest accumulated weight.

However, it was found that in case of this research it has a similar result to increasing the size of the IS window. Therefore, the authors decided that this added complexity does not improve results and it was not used for the results.

Individual strategies

Moving average crossover

Methodology. Individual strategies. MA crossover

- Aim: buy uptrend regimes.
- Logic: if fast_MA $>$ short_MA: Long; else: Neutral.
- IS window: 3Y
- OOS window: 1Y
- MA combinations:
  - Fast MA: 1,3,5,10,15,20,25,30,35,40,45,50,55,60,65.
  - Slow MA: 5, 10, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300.
- Exceptionally for this strategy, if Buy & Hold was better than all combinations of MA crossover, Buy & Hold is selected as the chosen combination (because the market trend was more efficient than any other combination)

Sell in May and go away

Methodology. Individual strategies. Sell in May and ...

- Aim: avoid worst performing months of the year.
- Logic: if month in (sell_month + sell_duration): Neutral; else: Long.
- IS window: 10Y
- OOS window: 1Y
- Parameter combinations:
  - Selling month: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
  - Selling duration: 1, 2, 3, 4, 5, 6.

ARIMA

ARIMA(1,1,1):

$$y_t = a_1 y_{t-1} + ... + a_p y_{t-p} + e_t + b_1 e_{t-1} + ... + b_q e_{t-q} \tag{12}$$

- Aim: predict next return based on a linear combination of previous returns, and on the error term of the last forecasted return.
- Logic: if predicted_price > (price+costs): Long; else if predicted_price < (price-costs): Short; else: Neutral
- IS window: 504D.
- OOS window: 1D.
- Parameters:
  - (p, d, q) = (1, 1, 1)
  - Slight fitting every day.
  - Fitted thoroughly every 63 days.

Macro-economic factor

Methodology. Individual strategies. Macro-economic factor

- Aim: generate investments signals based on the macro-economical situation of USA.
- Logic: if (ICSA_t - ICSA_t-1) < buy_quantile: Long; else if (ICSA_t - ICSA_t-1) > sell_quantile: Short; else: Neutral.
- IS window: 1Y.
- OOS window: 6M.
- Rollling window to calculate quantiles: 1260D (around 5Y).
- Parameter combinations:
  - Buying quantile: 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9.
  - Selling quantile: 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95.

Methodology. Individual strategies. Summary of crucial assumptions

Strategy	MA Crossover	Sell in May and go away	ARIMA model	Macro-economic factor
Logic definition	Strat. avoids downtrend markets	Strat. avoids worse performing months of the year	Strat. predicts next return based on a linear combination of previous returns, and on the error term of the last forecasted return	Strat. goes together with the macro-economic situation
Long signals	True	True	True	True
Neutral signals	True	True	True	True
Short signals	False	False	True	True
Long signals logic	if price >MA: Long	if month not in (sell_month + sell_duration): Long	if predicted_price > (price+costs): Long	if (ICSA_t - ICSA_t-1) < buy_quantile: Long
Neutral signals logic	if price <MA: Neutral	if month in (sell_month + sell_duration): Neutral	if ((price-costs) < (predicted_next_price) <(price+costs)): Neutral	if buy_quantile < (ICSA_t - ICSA_t-1) <sell_quantile: Neutral
Short signals logic	-	-	if predicted_price < (price-costs): Short	if sell_quantile < (ICSA_t - ICSA_t-1): Short

Methodology. Portfolio of strategies - Signal combination

The method for combining them is by calculating the mean from each signal individually.

- The overall portfolio will generate signals from -0.5 to $+1$ in S&P 500 futures, because two strategies can only generate a position of 0 (neutral) or 1 (long). The other two strategies can additionally generate a position of -1 (short).

Individual strategies

- Captures most of the upper trends and exit the long position when the two moving averages confirm a downtrend regime

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All Risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	0.45	18.19	56.78	14.04	7.17	0.07	10.4	0.14	0.59	1	0
MA crossover	8.16	**0.68**	**12.06**	**19.34**	**8.07**	**3.76**	**0.29**	**0.71**	**0.42**	**1.01**	172	2059

- Objective is not well accomplished as drawdowns do not happen in the same months, and the strategy is not able to stop them.

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	**0.45**	18.19	56.78	14.04	7.17	**0.07**	10.4	**0.14**	**0.59**	1	0
Sell in May	5.72	0.36	**15.71**	**48.12**	**12.25**	**7.11**	0.04	**6.58**	0.12	0.47	65	2371

Empirical results. Individual strategies. ARIMA

- This strategy performs better during periods of high volatility (2008-2009; 2020) and bad with lower volatility (during the 1990s; from 2003 to 2008; from 2009 to 2020).

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	**0.45**	18.19	56.78	14.04	**7.17**	**0.07**	10.4	0.14	**0.59**	1	0
ARIMA	7.31	0.42	**17.57**	**42.69**	**13.66**	8.37	0.07	**8.57**	**0.17**	0.54	3555	653

Empirical results. Individual strategies. Macro factor

- This strategy is better after 2000, when the negative correlation between ICSA and S&P 500 becomes stronger.

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	**0.45**	18.19	56.78	**14.04**	**7.17**	**0.07**	**10.4**	**0.14**	**0.59**	1	0
MacroFactor	3.2	0.19	**17.12**	**53.34**	14.59	8.77	0.01	11.69	0.06	0.22	1093	980

Portfolio of algorithmic strategies - Signal combination

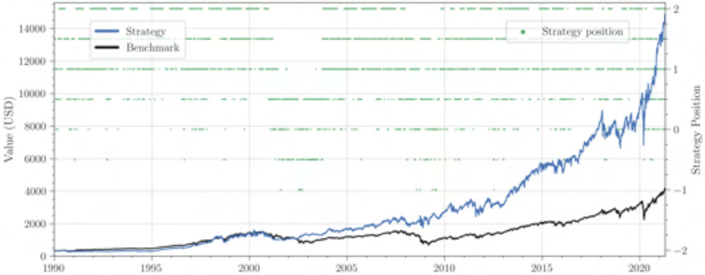
Empirical results. Portfolio of algorithmic strategies - Signal combination

- *MA crossover* strategy delivers very consistent results. It can go long during periods of upper trends (during the 1990s; from 2003 to 2008; and after 2009), and generally exits the market on periods of downtrends (from 2001 to 2003; during 2008; and in March of 2020).
- *Sell in May and go away* seems to generate random signals and spends less time being long on the underlying asset. This makes the strategy to be better than Buy & Hold strategy only during downtrends which last for more than one year (from 2000 to 2002; from 2008 to 2010).
- *ARIMA model* is better during periods of high volatility (2008-2009; 2020) and bad with lower volatility (during the 1990s; from 2003 to 2008; from 2009 to 2020).
- The *macro factor strategy* is better after 2000 when the negative correlation between ICSA and S&P 500 becomes stronger.

Empirical results. Portfolio of algorithmic strategies - Signal combination

- By diversifying the investment strategies we obtained stable results of the portfolio, as each of the strategies is better during different periods.

Empirical results. Portfolio of algorithmic strategies - Signal combination with 200% leverage



Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	8.21	0.45	18.19	56.78	14.04	7.17	0.07	10.4	0.14	0.59	1	0
Portfolio Lev. x1	6.81	0.61	**11.13**	20.01	8.08	4.71	0.21	0.85	0.34	0.84	1124	465
Portfolio Lev. x2	**12.68**	0.57	22.27	37.92	15.74	4.89	0.19	6.5	0.33	0.81	2248	465
MA crossover	8.16	**0.68**	12.06	**19.34**	**8.07**	**3.76**	**0.29**	**0.71**	**0.42**	**1.01**	172	2059
Sell in May	5.72	0.36	15.71	48.12	12.25	7.11	0.04	6.58	0.12	0.47	65	2371
ARIMA	7.31	0.42	17.57	42.69	13.66	8.37	0.07	8.57	0.17	0.54	3555	653
MacroFactor	3.2	0.19	17.12	53.34	14.59	8.77	0.01	11.69	0.06	0.22	1093	980

Individual strategies

Sensitivity Analysis. Individual strategies

Objective: test the robustness of all strategies by changing each of the optimization parameters one by one.

- Generally, each parameter was doubled and divided by 2, while others remained constant on each test.
- Additionally, the list of parameters tested on each IS was either shortened or lengthened.

Sensitivity Analysis. Individual strategies. MA crossover

- OOS window: it is changed from one year (OOS1Y) to six months (OOS6M) and two years (OOS2Y).
- IS window: it is changed from three years (IS3Y) to two years (IS2Y) and five years (IS5Y).
- List of parameters to be optimised on each IS window:
  - Original list (longlist) of parameters to be optimised on each IS window:
    - Fast MA: 1, 3, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65.
    - Slow MA: 5, 10, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300.
  - Newly tested list (shortlist) of parameters to be optimised on each IS window:
    - Fast MA: 5, 10, 20, 30, 40, 50
    - Slow MA: 50, 100, 150, 200, 250, 300

Sensitivity analysis. Individual strategies. MA crossover

Changing any parameter worsens risk-adjusted returns. However, all of them enter long positions at similar moments after drawdowns. Differences in performance are mainly caused by the moment that they exit buy positions when starting drawdowns.

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	0.45	18.19	56.78	14.04	7.17	0.07	10.4	0.14	0.59	1	0
longlist IS3Y OOS1Y	8.16	**0.68**	**12.06**	**19.34**	**8.07**	**3.76**	**0.29**	**0.71**	**0.42**	**1.01**	172	2059
longlist IS3Y OOS6M	7.98	0.66	12.19	**19.34**	8.19	**3.76**	0.27	0.72	0.41	0.98	134	1962
longlist IS3Y OOS2Y	6.82	0.53	12.75	36.77	9.17	6.55	0.1	2.81	0.19	0.74	116	1879
longlist IS2Y OOS1Y	7.18	0.57	12.71	21.37	9.17	4.02	0.19	1	0.34	0.78	103	1776
longlist IS5Y OOS1Y	5.54	0.37	14.86	51.9	10.39	6.87	0.04	5.51	0.11	0.53	71	1709
shortlist IS3Y OOS1Y	6.33	0.4	15.69	51.93	11.77	9.3	0.05	8.91	0.12	0.54	29	968

Sensitivity analysis. Individual strategies. Sell in May

- OOS window: it is changed from one year (OOS1Y) to six months (OOS6M) and two years (OOS2Y).
- IS window: it is changed from ten years (IS10Y) to five years (IS5Y) and twenty years (IS20Y).
- List of parameters to be optimised on each IS window:
  - Original list (shortlist) of parameters to be optimised on each IS window:
    - Selling month: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
    - Selling duration: 1, 2, 3, 4, 5, 6.
  - Newly tested list (longlist) of parameters to be optimised on each IS window:
    - Selling month: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
    - Selling duration: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Sensitivity analysis. Individual strategies. Sell in May

The strategy has very consistent risk-adjusted returns when changing each of the parameters. The best combination uses the longest IS period (20 years), which may suggest that, if any seasonality exists, it needs long training windows.

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	**0.45**	18.19	56.78	14.04	7.17	**0.07**	10.4	**0.14**	**0.59**	1	0
shortlist IS10Y OOS1Y	5.72	0.36	15.71	48.12	12.25	7.11	0.04	6.58	0.12	0.47	65	2371
shortlist IS10Y OOS6M	5.73	0.38	15.15	48.12	11.68	11.25	0.05	9.58	0.12	0.49	73	2671
shortlist IS10Y OOS2Y	5.52	0.36	15.37	48.12	11.71	**6.09**	0.04	**5.28**	0.11	0.47	65	2597
shortlist IS10Y OOS3Y	4.69	0.33	14.41	48.12	11.07	7.12	0.03	5.46	0.1	0.42	62	2601
shortlist IS5Y OOS1Y	5.19	0.33	15.92	54.08	12.48	17.8	0.03	19.13	0.1	0.42	69	2640
shortlist IS20Y OOS1Y	3.7	0.25	14.64	53.23	11.94	8.76	0.02	8.15	0.07	0.31	63	3132
longlist IS10Y OOS1Y	4.86	0.36	**13.35**	**39.5**	**9.5**	11.32	0.04	5.67	0.12	0.51	67	3978

Sensitivity analysis. Individual strategies. ARIMA

- Frequency of fitting of the model: it is changed from three months (fit63D) to:
  - one month (fit21D), and
  - six months (fit126D).
- Training window: it is changed from two years (train2Y) to:
  - one year (train1Y),
  - four years (train504D), and
  - hooked to the beginning of the data (hooked).

Sensitivity analysis. Individual strategies. ARIMA

The strategy obtains worse results after changing the training window length. However, it shows robustness in the parameter for the frequency of fitting the model. Also, the strategies behaves similarly during periods of high volatility of the market, which adds value to the overall portfolio.

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	**0.45**	18.19	56.78	14.04	**7.17**	**0.07**	10.4	0.14	**0.59**	1	0
train504D fit63D	7.31	0.42	17.57	**42.69**	13.66	8.37	**0.07**	**8.57**	**0.17**	0.54	3555	653
hooked fit63D	4.2	0.25	**16.98**	60.3	**13.49**	11.78	0.02	16.27	0.07	0.31	4871	1312
train1008D fit63D	4.45	0.25	17.5	65.19	14.35	9.28	0.02	15.18	0.07	0.31	3627	818
train252D fit63D	2.28	0.13	17.5	61.71	15.47	12.12	0	20.25	0.04	0.15	4065	779
train504D fit126D	6.55	0.37	17.61	46.91	13.67	8.48	0.05	9.57	0.14	0.48	3709	647
train504D fit21D	6.38	0.36	17.52	45.99	13.84	8.7	0.05	9.7	0.14	0.46	3487	667

Sensitivity analysis. Individual strategies. Macro factor

- Length of rolling window to calculate the distribution: it is changed from five years (1260D) to three years (756D) and ten years (2520D).
- OOS window: it is changed from six months (OOS6M) to three months (OOS3M) and one year (OOS1Y).
- IS window: it is changed from one year (IS1Y) to six months (IS6M) and two years (IS2Y).
- List of parameters to be optimised on each IS window:
  - Original list (shortlist) of parameters to be optimised on each IS window:
    - Buying quantile: 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9.
    - Selling quantile: 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95.
  - Newly tested list (longlist) of parameters to be optimised on each IS window:
    - Buying quantile: 0.1, 0.2, 0.3, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9.
    - Selling quantile: 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1.

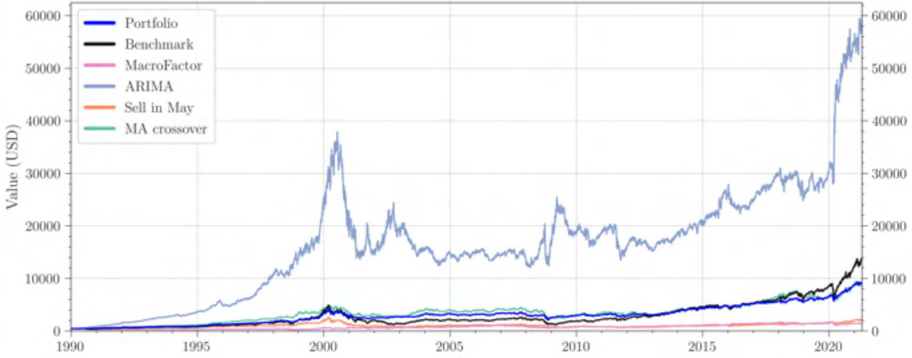
The strategy is robust to changes in default parameters. Risk-adjusted returns and risk metrics are very similar in all variations to the original strategy.

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	**8.21**	**0.45**	18.19	56.78	14.04	7.17	**0.07**	10.4	0.14	**0.59**	1	0
shortlist IS1Y OOS6M window1260D	3.2	0.19	17.12	53.34	14.59	8.77	0.01	11.69	0.06	0.22	1093	980
shortlist IS1Y OOS6M window756D	2.71	0.16	16.98	58.55	14.54	9.25	0.01	13.37	0.05	0.19	1061	1080
shortlist IS1Y OOS6M window2520D	3.52	0.21	17.11	52.39	14.19	8.75	0.01	11.13	0.07	0.25	1135	1120
shortlist IS1Y OOS3M window1260D	2.75	0.16	17.05	56.54	14.68	8.86	0.01	12.54	0.05	0.19	1183	1044
shortlist IS1Y OOS1Y window1260D	4.83	0.28	17.2	52.74	14.5	7.82	0.03	10.28	0.09	0.33	1043	803
shortlist IS6M OOS6M window1260D	3.2	0.18	17.34	55.93	14.71	8.64	0.01	12.33	0.06	0.22	1157	838
shortlist IS2Y OOS6M window1260D	3.64	0.21	16.98	53.21	13.87	7.95	0.01	9.97	0.07	0.26	1025	1089
longlist IS1Y OOS6M window1260D	4.7	0.32	**14.5**	**31.73**	**11.19**	**5.96**	0.05	**3.07**	**0.15**	0.42	925	2906

New underlying instrument - Nasdaq Composite

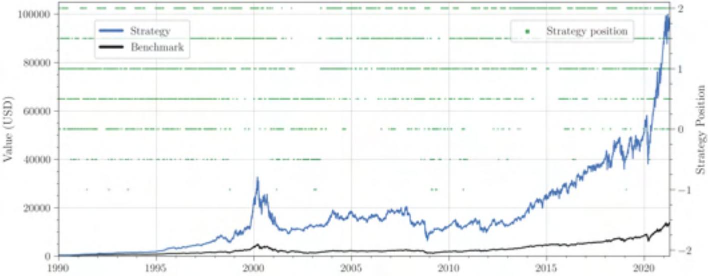
Sensitivity Analysis. New underlying - Nasdaq Composite

- Furthermore, this work analyzes the performance of the portfolio built from the algorithmic strategies on a different but correlated underlying asset, the NASDAQ Composite index.



Sensitivity Analysis. New underlying - Nasdaq Composite

- The use of leverage of 200% in case of the portfolio of diversified strategies enables us to increase return metrics within the limits of risk metrics on the level of our Benchmark.



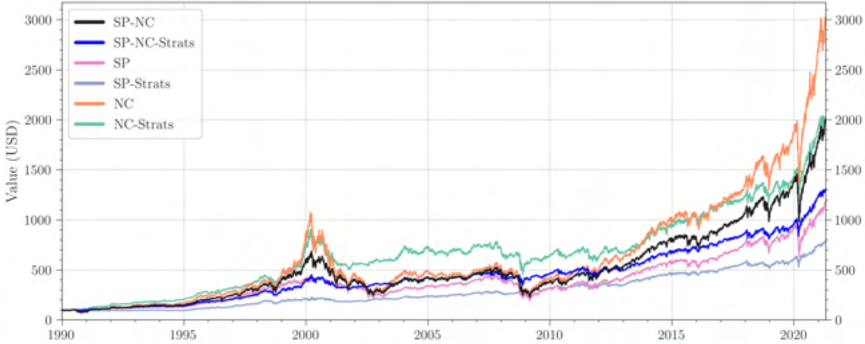
Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD	Num. trades	Neutral
B & H	11.47	0.49	23.57	77.34	19.02	15.09	0.07	52.32	0.15	0.6	1	0
Portfolio Lev. x1	10.07	0.7	**14.29**	**49.93**	**10.83**	14.23	0.14	**10.99**	0.2	0.93	1455	646
Portfolio Lev. x2	**18.7**	0.65	28.57	80.61	20.42	16.35	0.15	76.87	**0.23**	0.92	2910	646
MA crossover	9.9	0.59	16.65	61.23	12.7	16.35	0.1	21.17	0.16	0.78	587	2247
Sell in May	5.43	0.27	19.96	75.73	16.39	21.11	0.02	52.3	0.07	0.33	67	2636
ARIMA	16.62	**0.73**	22.83	71.05	16.85	19.67	**0.17**	53.78	**0.23**	**0.99**	4501	523
MacroFactor	4.08	0.19	21.55	53.08	19.4	**9.89**	0.01	21.96	0.08	0.21	1065	1388

Ensemble model for S&P500 and Nasdaq

Sensitivity Analysis. Ensemble model for S&P500 and Nasdaq

Objective: add one extra layer of diversification by building an ensemble model that produces an investment position on each index every day.

- Each model of strategies is applied individually to each index.
- The portfolio is rebalanced every day, and each model has a weight of 50% on each index.



Sensitivity Analysis. Ensemble model for S&P500 and Nasdaq

Again, having such risk metrics allows us to add leverage to the portfolio and have a similar level of risk to the Buy & Hold strategy with daily rebalancing.

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk	ARC MD	ARC AMD
SP NC	10.03	0.5	20.19	66.98	16.24	13.73	0.07	30.15	0.15	0.62
SP NC Strats Lev. x1	8.54	**0.71**	11.96	31.8	8.78	6.62	0.19	2.21	0.27	**0.97**
SP NC Strats Lev. x2	**16.14**	0.67	23.92	55.86	16.91	7.31	0.19	16.51	0.29	0.95
SP	8.21	0.45	18.19	56.78	14.04	7.17	0.07	10.4	0.14	0.59
SP Strats	6.81	0.61	**11.13**	**20.01**	**8.08**	**4.71**	**0.21**	**0.85**	**0.34**	0.84
NC	11.47	0.49	23.57	77.34	19.02	15.09	0.07	52.32	0.15	0.6
NC Strats	10.07	0.7	14.29	49.93	10.83	14.23	0.14	10.99	0.2	0.93

Research Hypotheses Verification

First Hypothesis:

the market price reflects all public information, and therefore, it is impossible to obtain higher risk-adjusted returns than the market itself -> **rejected**

Second Hypothesis:

whether it is possible to obtain better risk-adjusted returns by combining signals from several investment strategies, than from each of them individually. -> **it is possible**

Third Hypothesis:

whether it is possible to obtain better risk-adjusted returns by combining signals from more than one basis instrument instead of only one -> **it depends on the difference of risk-adjusted return metrics between the strategies for these instruments**

Fourth Hypothesis:

whether methodological advancement (robust optimization criterion and intraperiod selection of optimal parameters can results in better risk-adjusted returns in out-os-sample period) -> **based on the research described in this paper we can say that they can**

Fifth Hypothesis:

whether the final results are robust to initial selction of parameters -> **they are robust**

Conclusions

- In the process of construction, estimation and optimization of algorithmic investment strategies we have found that there is a need:
  - to design a robust Walk-Forward procedure of testing divided on multiple in-sample and out-of-sample periods,
  - to construct robust optimization criteria,
  - to select intra-period optimal parameters,
- In order to increase risk-adjusted return metrics form single strategies we have to:
  - combine multiple investment strategies based on various theoretical concepts of Buy/Sell signals,
  - construct these strategies based on basis instruments selected from various asset classes,
  - diversify the portfolio of investment strategies through the use of various frequency of data,

Research extensions

- to repeat this research with larger set of individual classical strategies
- to add algorithmic investment strategies (AIS) based on ML
- to add instruments from various types of asset classes (commodities, currencies, bonds, volatility, real-estates, hedge funds, cryptocurrencies,)
- to add AIS based on HF data, not only daily frequency

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Results of investment strategies after publication

2021-04-26 - 2021-10-16

Results after publication - S&P 500

Benchmark, individual strategies and combined portfolio with leverage = 100%:

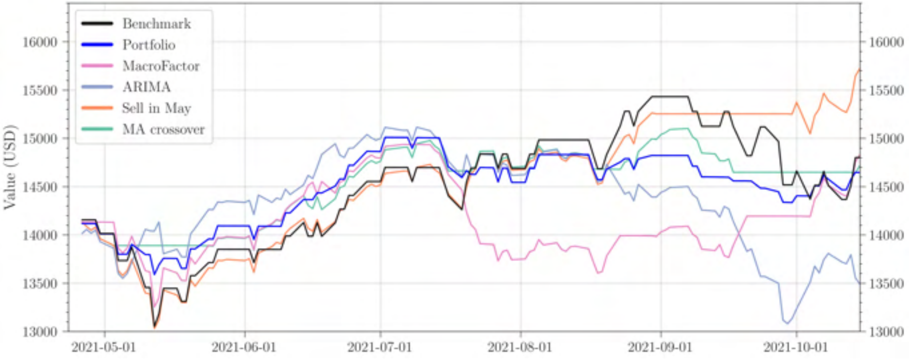
Results after publication - S&P 500

Benchmark and combined portfolio with leverage = 200%:

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk *100	ARC MD	ARC AMD	Num. trades	Neutral
B & H	14.92	1.31	11.36	5.21	5.21	**0.11**	3.76	0.35	2.86	2.86	1	0
Portfolio Lev. x1	5.4	0.79	**6.83**	3.68	3.68	0.26	1.16	0.24	1.47	1.47	33	9
Portfolio Lev. x2	10.56	0.77	13.67	**7.3**	**7.3**	0.26	1.12	1.87	1.45	1.45	66	9
MA crossover	3.02	0.41	7.37	3.86	3.86	0.17	0.32	**0.18**	0.78	0.78	23	38
Sell in May	**19.66**	**2.11**	9.33	4.01	4.01	0.18	**10.35**	0.27	**4.91**	**4.91**	3	43
ARIMA	-10.7	-0.97	11.05	9.5	9.5	0.16	-1.09	1.61	-1.12	-1.12	119	5
MacroFactor	9.44	0.83	11.37	7.33	7.33	0.26	1.07	1.57	1.29	1.29	19	1

Results after publication - Nasdaq Composite

Benchmark, individual strategies and combined portfolio with leverage = 100%:

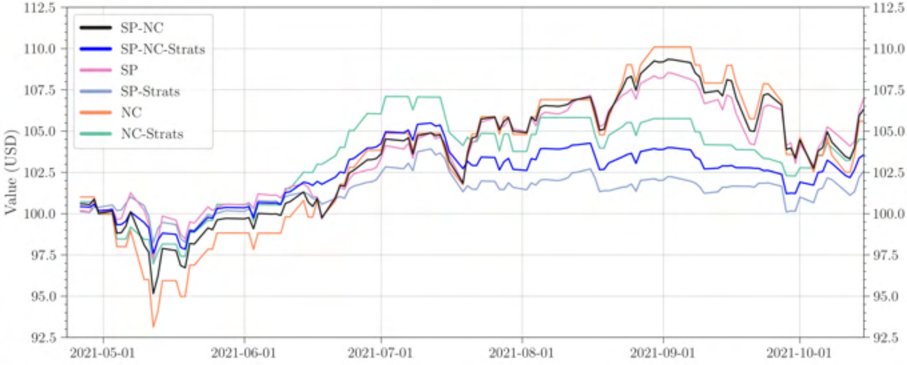


Benchmark and combined portfolio with leverage = 200%:

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk *100	ARC MD	ARC AMD	Num. trades	Neutral
B & H	11.91	0.79	15.12	7.8	7.8	**0.15**	1.2	1.33	1.53	1.53	1	0
Portfolio Lev. x1	9.52	1.09	8.75	4.5	4.5	0.28	2.3	0.49	2.12	2.12	33	6
Portfolio Lev. x2	19.03	1.09	17.5	8.93	8.93	0.28	2.32	3.86	2.13	2.13	66	6
MA crossover	8.23	0.98	**8.4**	**2.97**	**2.97**	**0.15**	2.72	**0.11**	2.77	2.77	11	42
Sell in May	**26.94**	**1.98**	13.6	7.8	7.8	**0.15**	**6.84**	1.2	**3.45**	**3.45**	3	21
ARIMA	-6.99	-0.48	14.5	14.13	14.13	0.27	0.24	7.85	-0.49	-0.49	113	4
MacroFactor	7.65	0.57	13.37	7.8	7.8	0.25	0.56	2.05	0.98	0.98	21	25

Results after publication - Ensemble model

Benchmarks, combined portfolios applied to each index, and ensemble model of both portfolios with leverage = 100%:



Results after publication - Ensemble model

Benchmarks, and ensemble model of both portfolios with leverage = 200%:

Strategy	ARC	IR*	aSD	MD	AMD	MLD	IR**	All risk * 100	ARC MD	ARC AMD
B & H	13.5	1.06	12.76	6.04	6.04	0.12	2.36	0.57	2.23	2.23
SP NC Strats Lev. x1	7.48	1.02	7.3	4.05	4.05	0.26	1.89	0.31	1.85	1.85
SP NC Strats Lev. x2	14.91	1.02	14.61	8.03	8.03	0.26	1.9	2.42	1.86	1.86
SP	**14.92**	**1.31**	11.36	5.21	5.21	**0.11**	**3.76**	0.35	**2.86**	**2.86**
SP Strats	5.4	0.79	**6.83**	**3.68**	**3.68**	0.26	1.16	**0.24**	1.47	1.47
NC	11.91	0.79	15.12	7.8	7.8	0.15	1.2	1.33	1.53	1.53
NC Strats	9.52	1.09	8.75	4.5	4.5	0.28	2.3	0.49	2.12	2.12

Thank you for your attention!

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