VERIFICATION OF INVESTMENT OPPORTUNITIES ON THE CRYPTOCURRENCY MARKET WITHIN THE MARKOWITZ FRAMEWORK

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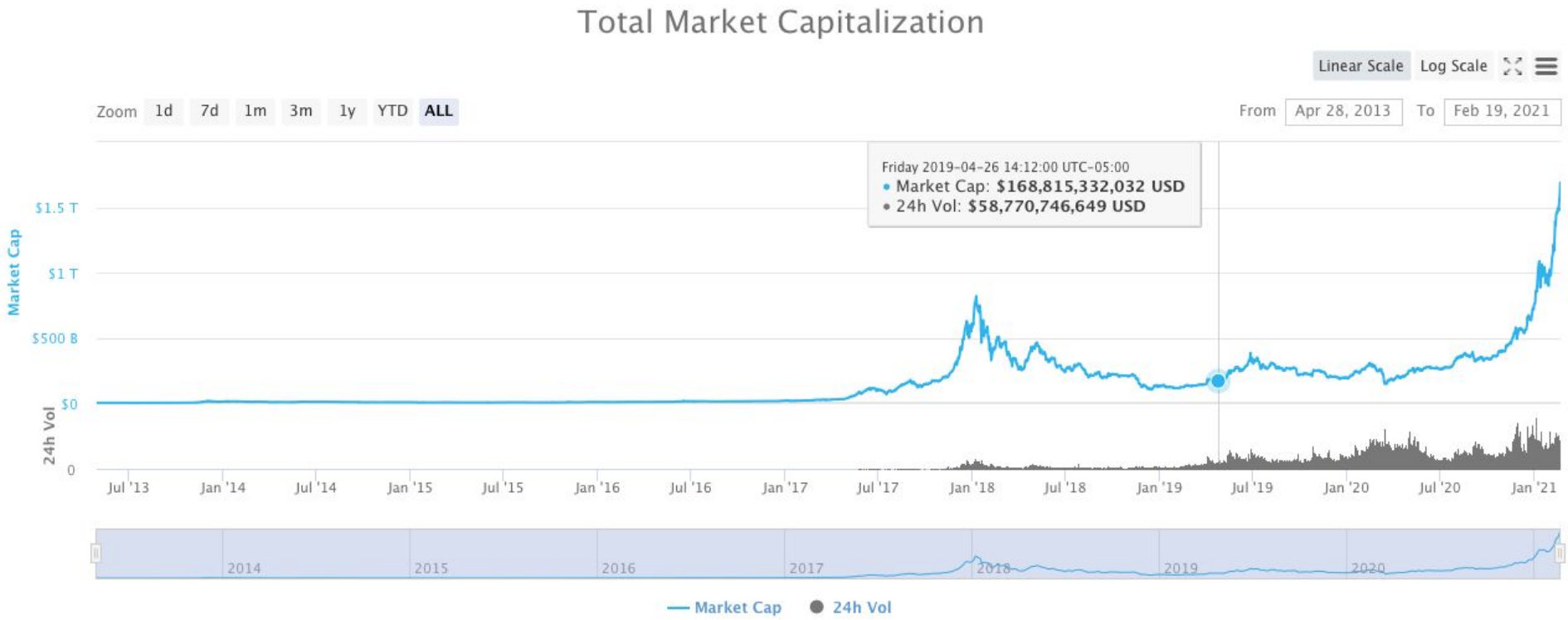
Purpose of the research

- **Testing efficiency** of the dynamic **investment portfolio** created of solely **cryptocurrencies** within the **Markowitz framework**.
- Finding the **best** combination of **optimization parameters** for the **profitable portfolio.**
- **Comparing** portfolio profitability with **benchmarks**.
- **Verifying robustness** of the portfolio **performance** to the **initial assumptions** about strategy parameters.

Why Crypto?

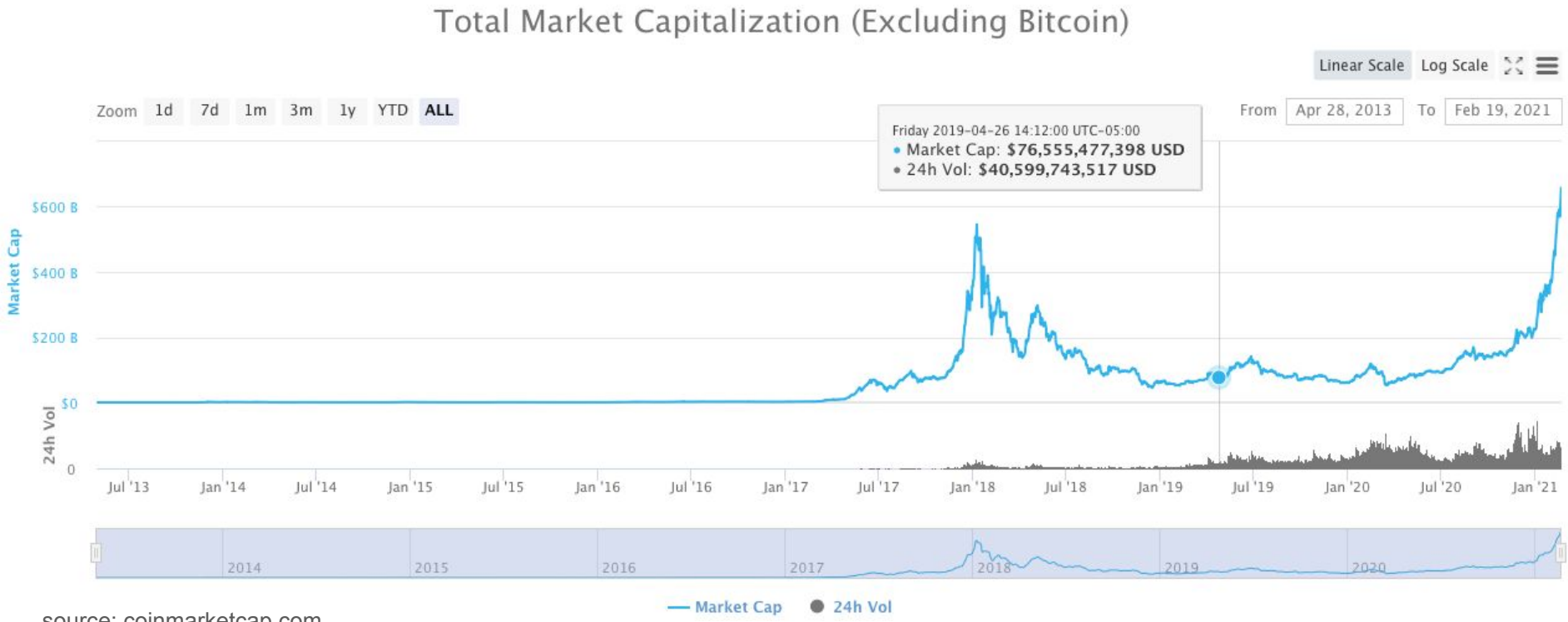
- A **novel, separate** and **alternative** class of investable financial instrument.
- **Higher volatility**, which gives opportunity for **higher profits** but also **higher losses**!
- Relatively **low correlation** with traditional assets on regulated markets**,** which increases **diversification** possibilities.
- **Flexibility of investing** - crypto market is open 24/7, no time frame restrictions.
- **Security** - blockchain transactions are encrypted, signed by a private key and verified by a public key.
- Low **entry threshold** - cryptocurrencies are available to small private investors.

Why Crypto?



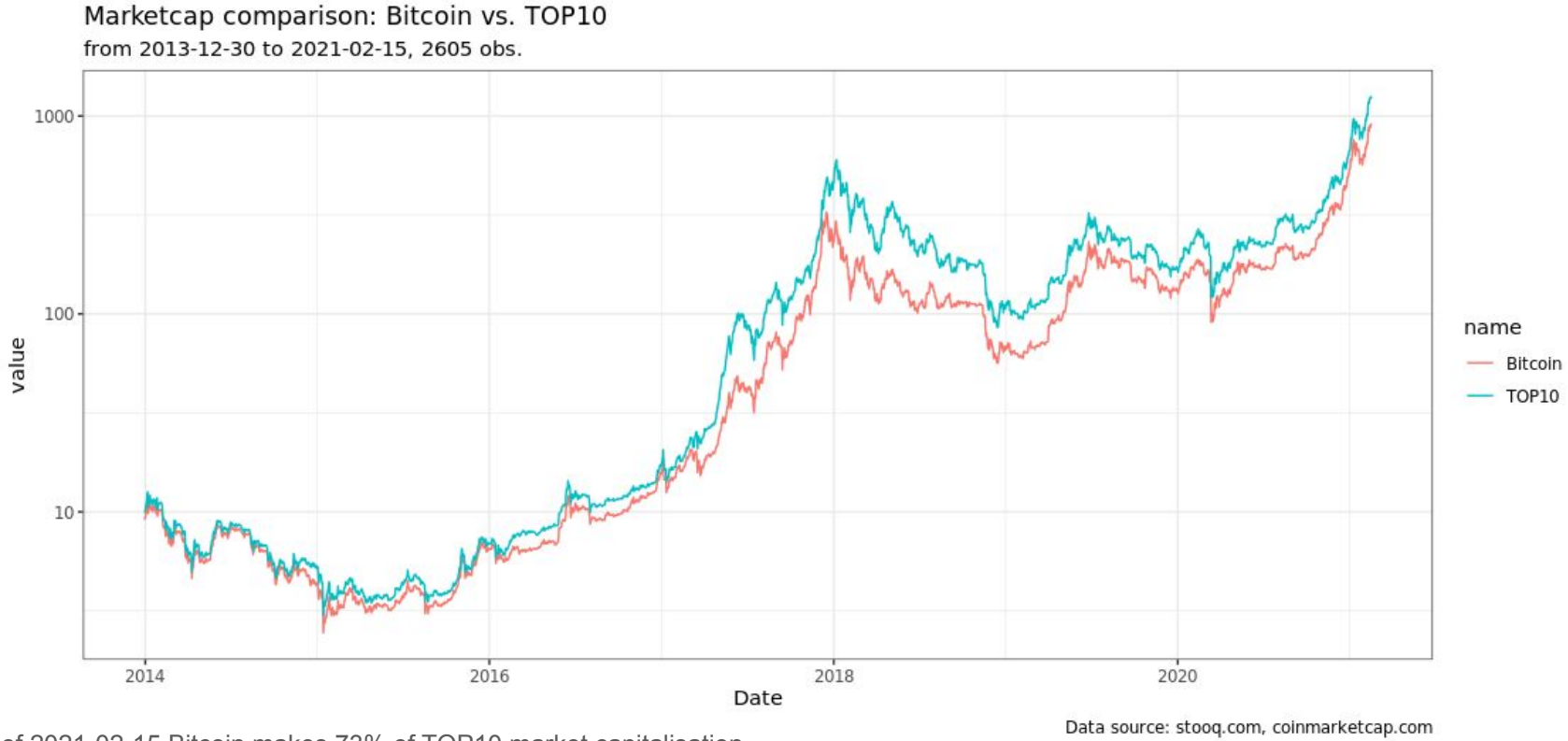
source: coinmarketcap.com

Why Crypto?



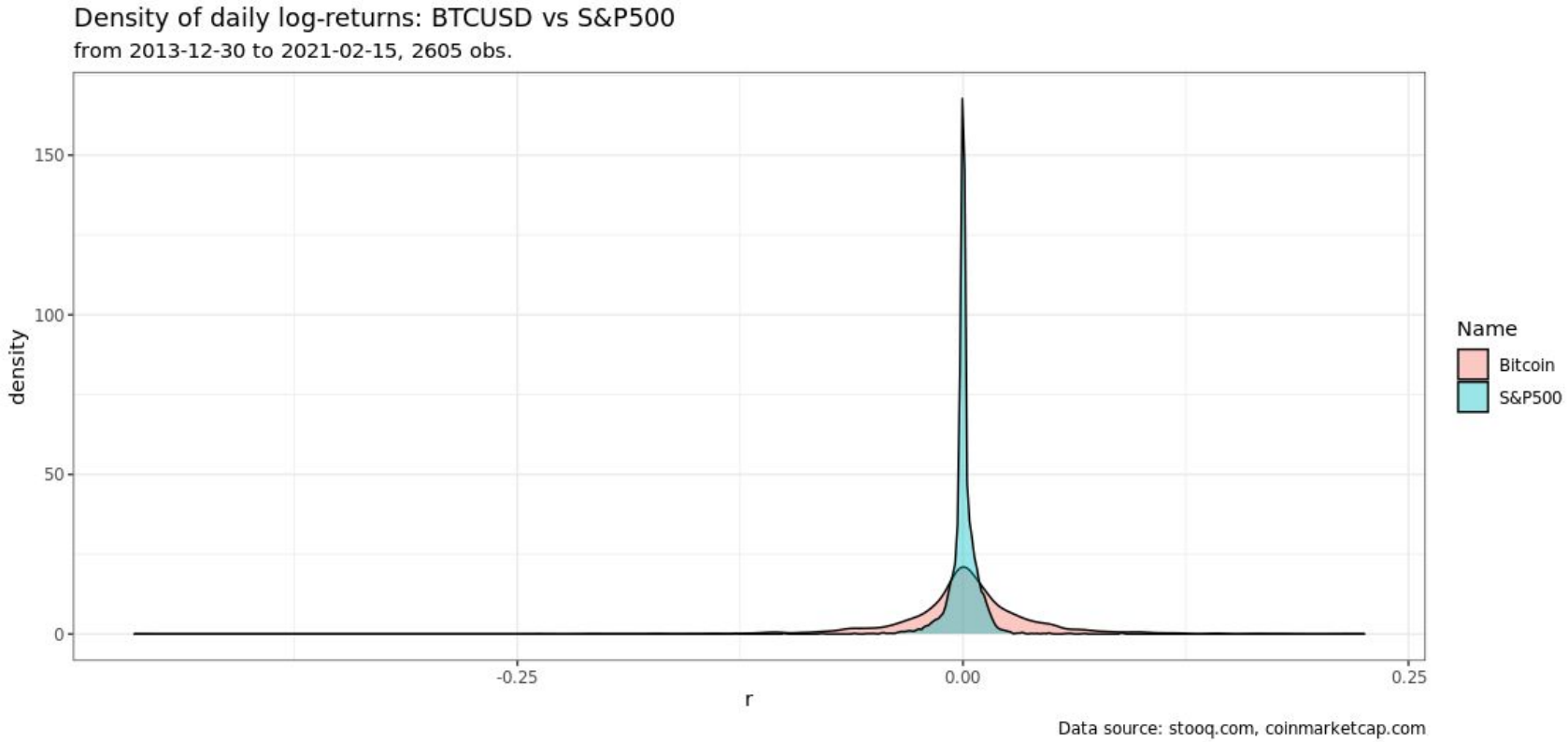
source: coinmarketcap.com

Comparison of market capitalisation



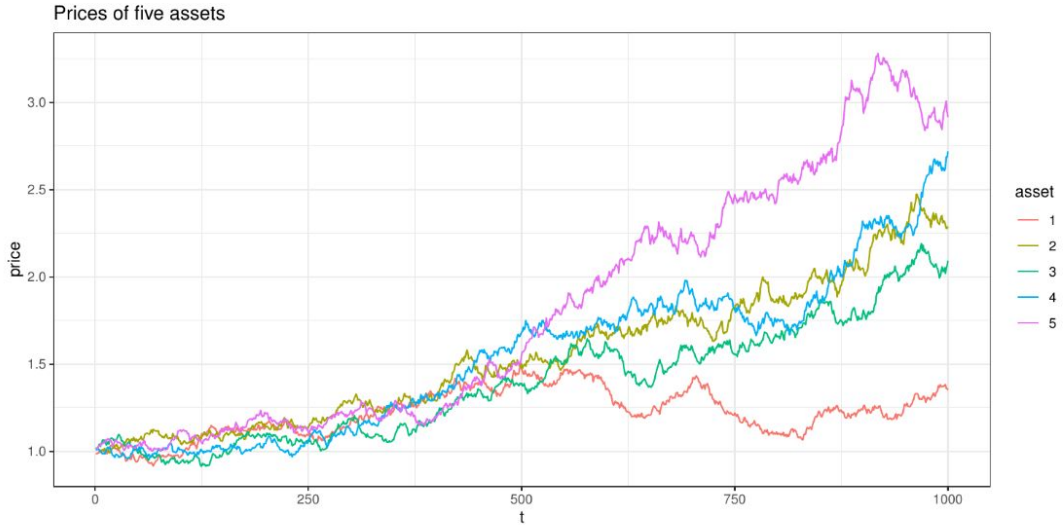
As of 2021-02-15 Bitcoin makes 73% of TOP10 market capitalisation.

Returns volatility: Bitcoin vs S&P500



Risk is important

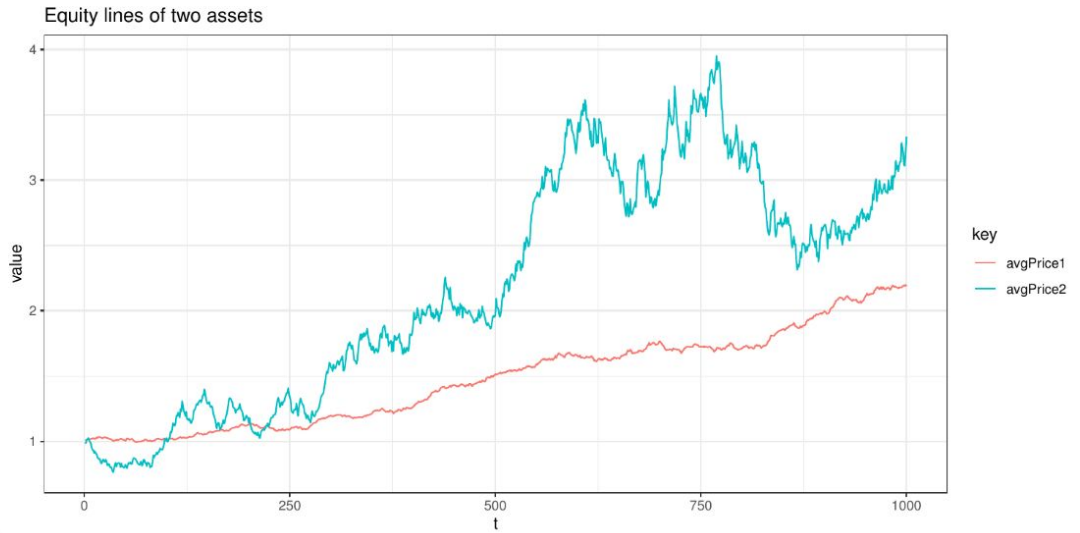
- Suppose we consider building a portfolio from the specified n assets:



- Optimal asset allocation problem: how to select values of the weights vector $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)$?

Risk is important

- Which optimization criterion to choose? Maximum return? Minimum risk, measured by portfolio return variance? Maximum risk/variance ratio?



- Which equity line would you prefer?

Investment risk diversification

- Investing into different assets can significantly **reduce risk**.
- Example: Investing into two assets R_1, R_2 with the same expected return and the same variance instead of investing only in one of them.

$$\mathbb{E}(\frac{1}{2}R_1 + \frac{1}{2}R_2) = \mathbb{E}R_1 = \mathbb{E}R_2$$

$$\mathrm{Var}(\frac{1}{2}R_1 + \frac{1}{2}R_2) = \mathrm{Var}(\frac{1}{2}R_1) + \mathrm{Var}(\frac{1}{2}R_2) + 2\,\mathrm{Cov}(\frac{1}{2}R_1, \frac{1}{2}R_2)$$

$$\mathrm{Var}(\frac{1}{2}R_1 + \frac{1}{2}R_2) = \frac{1}{2}\,\mathrm{Var}(R_1) + \frac{1}{2}\,\mathrm{Corr}(R_1, R_2)\,\mathrm{Var}(R_1) \leqslant \mathrm{Var}(R_1)$$

- The same expected return with lower variance!
- The lower $\mathrm{Corr}(R_1, R_2)$, the lower risk!

How about investing in many assets?

- We can allocate money in numerous assets from different markets, including crypto!
- Anyway, **what part of capital** should be allocated in each asset?
  - Simply just put **the same** amount of money in each asset?
  - Use weights which are proportional to **market capitalization**?
  - Or **optimize assets weights** using expected returns and variance-covariance matrix in a way reflecting your risk appetite?

Markowitz Portfolio Theory

There is a **trade-off** between market risk (variance)
and expected return of the portfolio

The investors decide whether to ***maximize*** expected **return**
or ***minimize*** portfolio **risk**.

Not only types of assets investor includes into portfolio matter,
but the **relationship between** these assets as well!

The **optimal weights** assigned to instruments in the portfolio
would depend on **correlation** between them

Markowitz Portfolio Theory

The **less correlation** there exists in the portfolio, the **more diversified** it is!

The concept of **diversification** has been coined by Markowitz (1952).
It proved to be one of most important tools for **risk reduction**
and portfolio **performance improvement**.

The **variance-covariance matrix** is estimated and CML line obtained,
which incorporates all effective portfolios to satisfy the risk-return
preferences of the investor.

Basics of portfolio theory

- Multivariate returns of available assets are represented by the random vector $\boldsymbol{R} = (R_1, \ldots, R_n)$ with the expected value vector $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$.
- Risk-free asset with expected return μ_0 and returns variance equal to 0.
- Portfolio percentage weights:

$$w_0 \text{ and } w = (w_1, \ldots, w_n) \in \mathbb{R}^n :$$

$$w_0 + \sum_{i=1}^{n} w_i = 1,$$

$$\forall i \in \{1, \ldots, n\} \;\; w_i \geqslant 0$$

Basics of portfolio theory

- Expected return from portfolio:

$$w_0 \mu_0 + w^T \mu = \sum_{i=0}^{n} w_i \mu_i$$

- Variance of portfolio returns:

$$w^T \Sigma w = \sum_{i=1}^{n} w_i^2 \operatorname{Var}(R_i) + \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} w_i w_j \operatorname{Cov}(R_i, Rj)$$

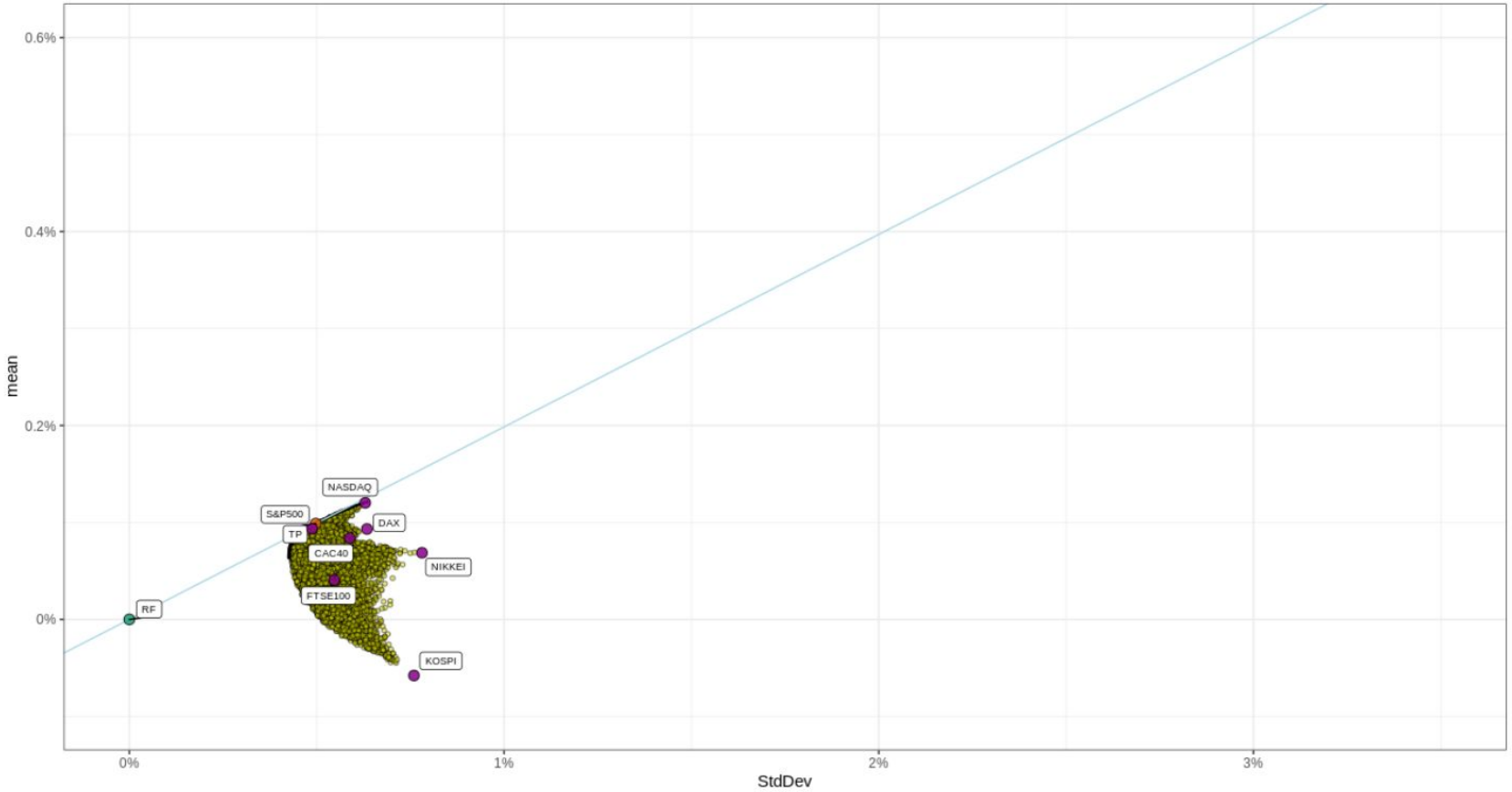
The Markowitz Problem

- A rational investor wants expected **returns as high** as possible and returns **variance as low** as possible. However, these goals are at odds!
- We can choose **target expected return** and **minimize return variance**.
- The target depends on **preferences** of the investor.
- Numerical methods are used to find the vector of **optimal weights** w^*.

$$\begin{cases} \min \ w^T \Sigma w \\ w_0 \mu_0 + w^T \mu = \rho \\ w_0 + \sum_{i=1}^{n} w_i = 1 \\ \forall_{i \in \{1,\ldots,n\}} w_i \geqslant 0 \end{cases}$$

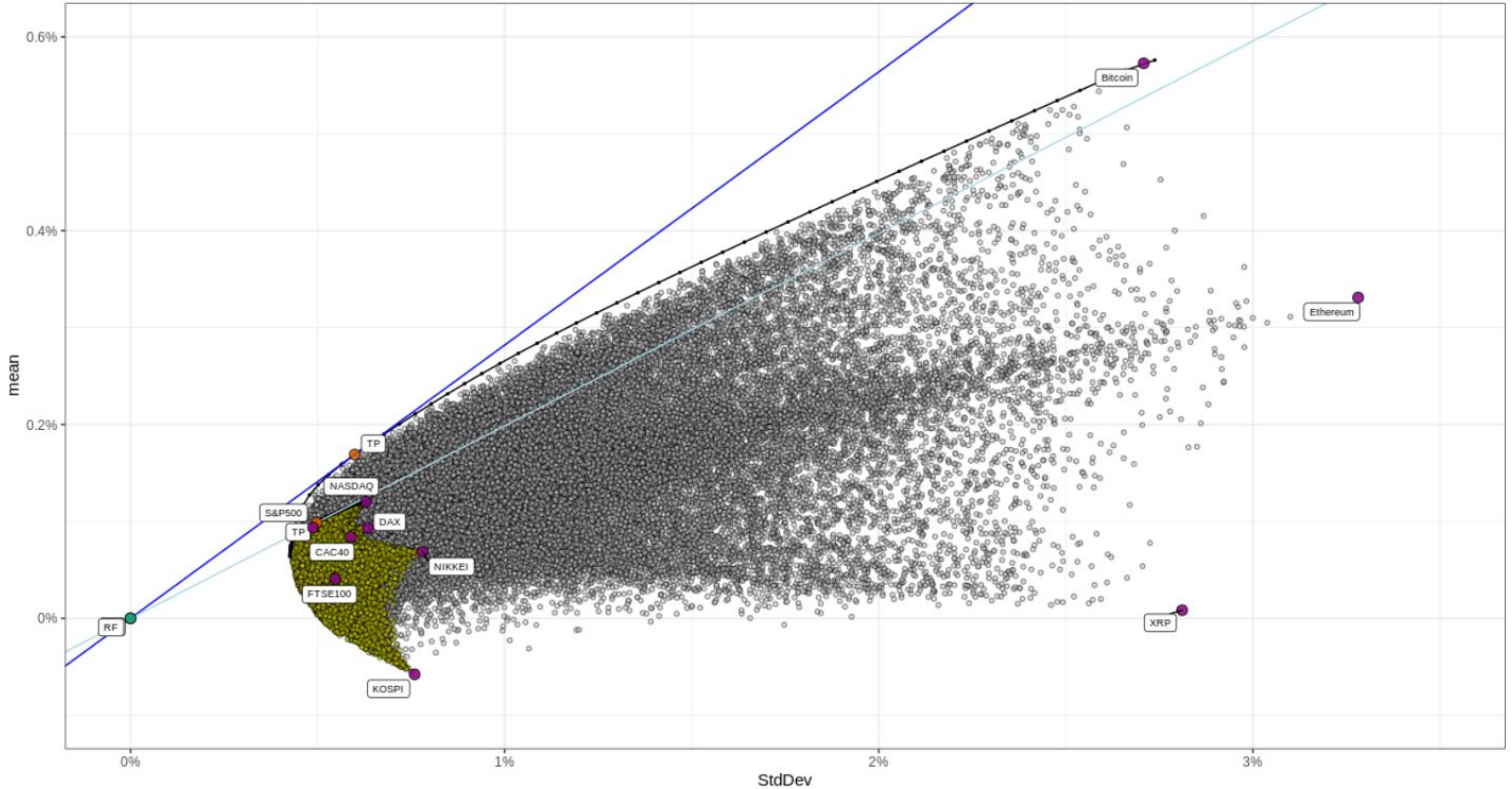
- **Crucial question:** how to get the variance-covariance Σ matrix?

Possible portfolios: equity indices only



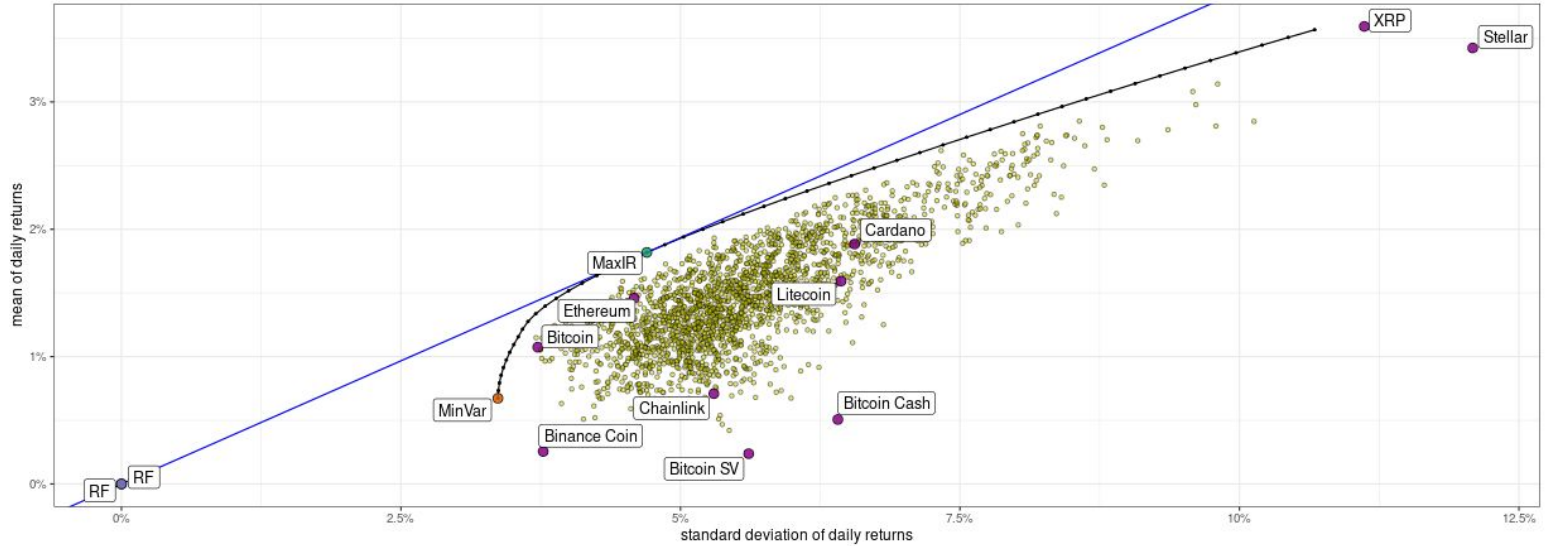
The CML line represents all effective portfolios, which are linear combinations of a tangency portfolio and a risk-free asset.

Including cryptocurrencies



The new tangency portfolio is more effective! The new CML line is substantially steeper!

In this study: only cryptocurrencies!



Maximum IR portfolio structure:

	(%)
Bitcoin	48
Ethereum	27
XRP	15
Stellar	11

Minimum Variance portfolio structure:

	(%)
Bitcoin	51
Binance Coin	48

Efficient frontier on 2020-12-01, lookback 30 days

Practical investment challenges

However, there are important questions!

- Which method to produce **forecasts of variance-covariance** matrix?
- What **estimation window** for estimation of variance-covariance matrix?
- How often to **re-estimate weights** and rebalance our position?
- What is our **risk profile**? How much money to invest in risky assets and how much in a risk-free asset?
- Is the **expected return** and **variance** sufficient to assess risk and strategy performance?

Answers to these questions can be provided with the backtesting procedure on historical observations for different parameters and assumptions.

Data description and filtering

Step 1. **Download** available data for all existing cryptocurrencies from coinmarketcap.com including close price, market capitalization, date and name.

Step 2. The start date of the data for this research was restricted to begin from 2014-01-01 and end up on 2019-04-26. Hence, the total number of observations is 1941 days (weekends included).

Step 3. Coins with observations less than 60 days were **excluded**.

Step 4. **Simple daily returns** for the last month and rolling standard deviation with the window of 30 days back were calculated.

Step 5. The **rank** in terms of market capitalization is created for each day.

Data description and filtering

Filtering method I:

Choose TOP100 highest market capitalization (marketcap) coins for every rebalancing date (RB) and then segregate TOP10 with maximum return for last 30 days and/or TOP10 with minimum standard deviation (SD) for last 30 days.

Filtering method II:

Choose TOP30 highest marketcap coins for every RB date, then pick top 10 with maximum return and/or TOP10 with minimum SD (as above).

Filtering method III:

Choose TOP10 highest marketcap for every RB date.

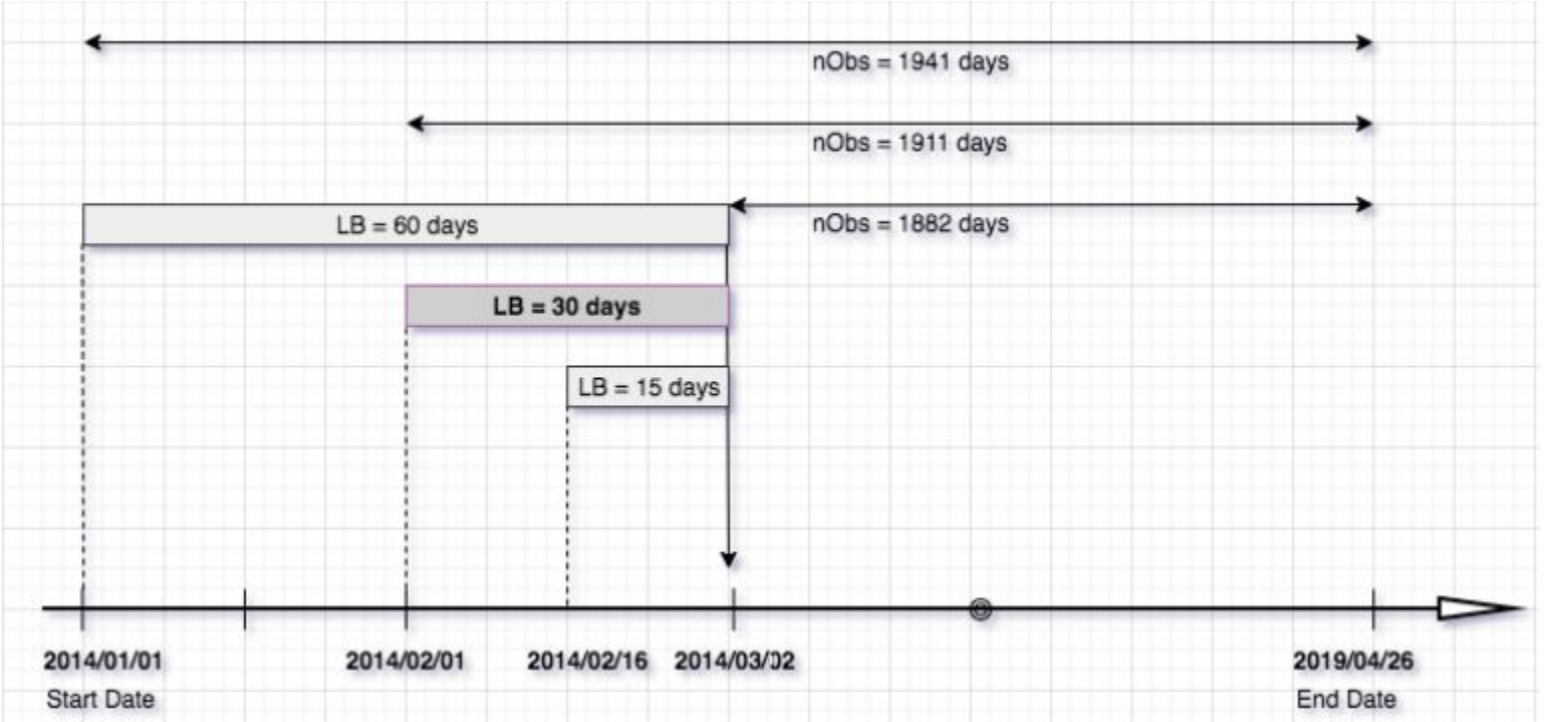
Construction of a dynamic portfolio strategy

- The initial date of each strategy is set to **2014-03-02**, whereas the last day is set to **2019-04-26**.
- Hence, the length of portfolio holding period is **1882** days.
- At every rebalancing (**RB**) date TOP10 marketcap coins are chosen (Filtering Method III), assuming they have been observed at least for 30 days.
  Then we:
  - **estimate** expected values of **returns** vector and **variance-covariance** matrix Σ,
  - **optimize weights** following your risk profile,
  - **construct** the **portfolio** - allocate money proportionally to weights (long only positions),
  - wait for the **next rebalancing** moment.

Construction of a dynamic portfolio strategy

- **Rebalancing frequency** (**RB**) is set to **14 days** due to high volatility
  (RB is also subject to sensitivity analysis)
- **Lookback period** (**LB**) or number of historical observations used to estimate variance-covariance matrix is set to **30 days**, because of high volatility and low level of predictability of digital coins.
  (LB is also subject to sensitivity analysis)
- As a result, **52 coins** appeared in the portfolio rebalancing **at least once**.

Portfolio creation:(LB) parameter analysis for optimization



Source: own calculations based on period 2014/03/02 – 2019/04/26. nObs is the number of observations in days, LB is the lookback period or number of historical observations used to estimate variance-covariance matrix.

Portfolio creation: Objectives and Restrictions

Objective 1: maximum Information Ratio (**IR**) - the measure of the risk-adjusted portfolio return

Objective 2: mininum variance (**MV**) of portfolio daily returns.

Restrictions:

- full investment -> sum of weights equal to 1,
- long only restriction -> no negative weights,
- minimum weight is 1% and maximum weight is 60% to prevent Bitcoin from taking over the whole portfolio,
- transactional cost is 1% (TC is subject to sensitivity analysis).

Parameters combination for the Markowitz optimization

The number of parameters combinations used for Markowitz optimization and hence **number of tested strategies** is **2*3*3*2 = 36**:

- **Optimization objective**: maximum Information Ratio (**IR**), minimum variance (**MV**) (2 values)
- **Lookback period** (**LB**) based on which returns and variance-covariance matrix are estimated: 15 days, **30 days**, 60 days (3 values)
- **Rebalancing period** (**RB**), period after which new weights for cryptocurrencies are calculated (RB): 7 days, **14 days**, 21 days (3 values)
- **Transaction costs** (**TC**): 1% and 2% (2 values)

Parameters combination for Markowitz optimization in the study

The **primary combinations** of initially assumed parameters for Markowitz optimization then would be the following:

- MarkCap10_LB30_RB14_TC1_IR
- MarkCap10_LB30_RB14_TC1_MV
- MarkCap10_LB30_RB14_TC2_IR
- MarkCap10_LB30_RB14_TC2_MV

Benchmarks

All 36 strategies will be applied to benchmarks defined below, portfolio performance and equity lines will reveal the outcomes of comparison MPT with benchmark investments

1. S&P500 index buy-and-hold (SPX)
2. Bitcoin buy-and-hold (BTC_Single)
3. Market capitalization weighted portfolio (MarkCap)
4. Equally weighted portfolio (EqualWeights), where each coin gets weight of 10%

Performance measures

- Annualized Return Compounded (aRC)

$$aRC = \left(\left(1 + \frac{P_{i,T}}{P_{i,0}}\right)^{\frac{365}{T}} - 1\right) * 100,$$

where $P_{i,T}$ is a price of i-th asset at the end of interval T

- Annualized Standard Deviation of daily returns (aSD)

$$aSD = \sqrt{\frac{365}{T}\sum_{t=1}^{T}(r_{i,t} - \bar{r})^2} \; *100$$

- Information Ratio (IR), which describes the relation of the portfolio annualized rate of return to the annualized volatility of the return (IR)

$$IR = \frac{aRC}{aSD}$$

Performance measures

- Maximum Drawdown (MD), which is the maximum loss from a peak to a minimum of a portfolio before a new peak is attained

$$MD(T) = max_{\tau \in [0,T]}(max_{t \in [0,\tau]} R_{i,T}^{(p)} - R_{i,\tau}^{(p)}) * 100$$

- the relation of Information Ratio to Maximum Drawdown (IRMD)

$$IRMD = \frac{IR}{MD}$$

- the relation of product of IR and Annualized Return Compounded to the Maximum Drawdown (IRaRCMD)

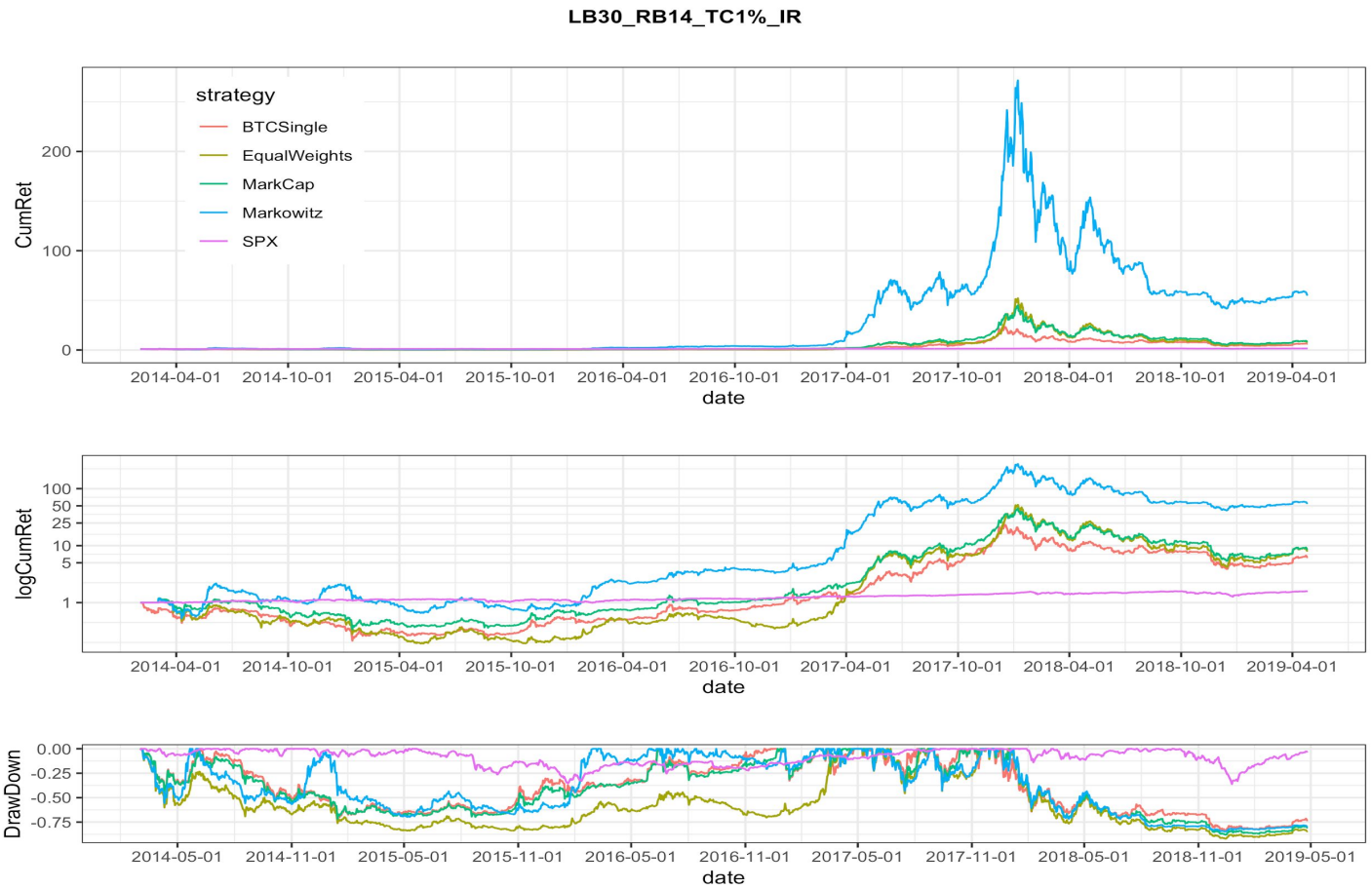
$$IRaRCMD = IR * \frac{aRC}{MD}$$

RESULTS, 2014/03/02 - 2019/04/26

Table 1: Performance measures of 4 strategies with initial set of parameters

	aRC	aSD	MD	IR	IRMD	IRaRCMD	nObs
MarCap10_LB30d_RB14_TC1_IR							
Markowitz	118.3	81.9	84.7	1.44	1.70	2.02	1882
EqualWeights	50.4	86.9	91.9	0.58	0.63	0.32	1882
MarkCap	52.4	72.6	87.9	0.72	0.82	0.43	1882
MarCap10_LB30d_RB14_TC1_MV							
Markowitz	69.0	77.1	82.1	0.90	1.09	0.75	1882
EqualWeights	50.4	86.9	91.9	0.58	0.63	0.32	1882
MarkCap	52.4	72.6	87.9	0.72	0.82	0.43	1882
MarCap10_LB30d_RB14_TC2_IR							
Markowitz	64.3	83.5	89.5	0.77	0.86	0.55	1882
EqualWeights	48.3	86.9	92.0	0.56	0.60	0.29	1882
MarkCap	50.7	72.6	88.1	0.70	0.79	0.40	1882
MarCap10_LB30d_RB14_TC2_MV							
Markowitz	54.1	81.3	89.4	0.67	0.75	0.40	1882
EqualWeights	48.3	86.9	92.0	0.56	0.60	0.29	1882
MarkCap	50.7	72.6	88.1	0.70	0.79	0.40	1882
BTC SPX							
BTC_Single	54.5	73.5	83.4	0.74	0.89	0.48	1882
SPX	9.5	13.2	19.8	0.72	3.62	0.34	1298

Figure 1: Visualization of results for scenario LB30_RB14_TC1_IR



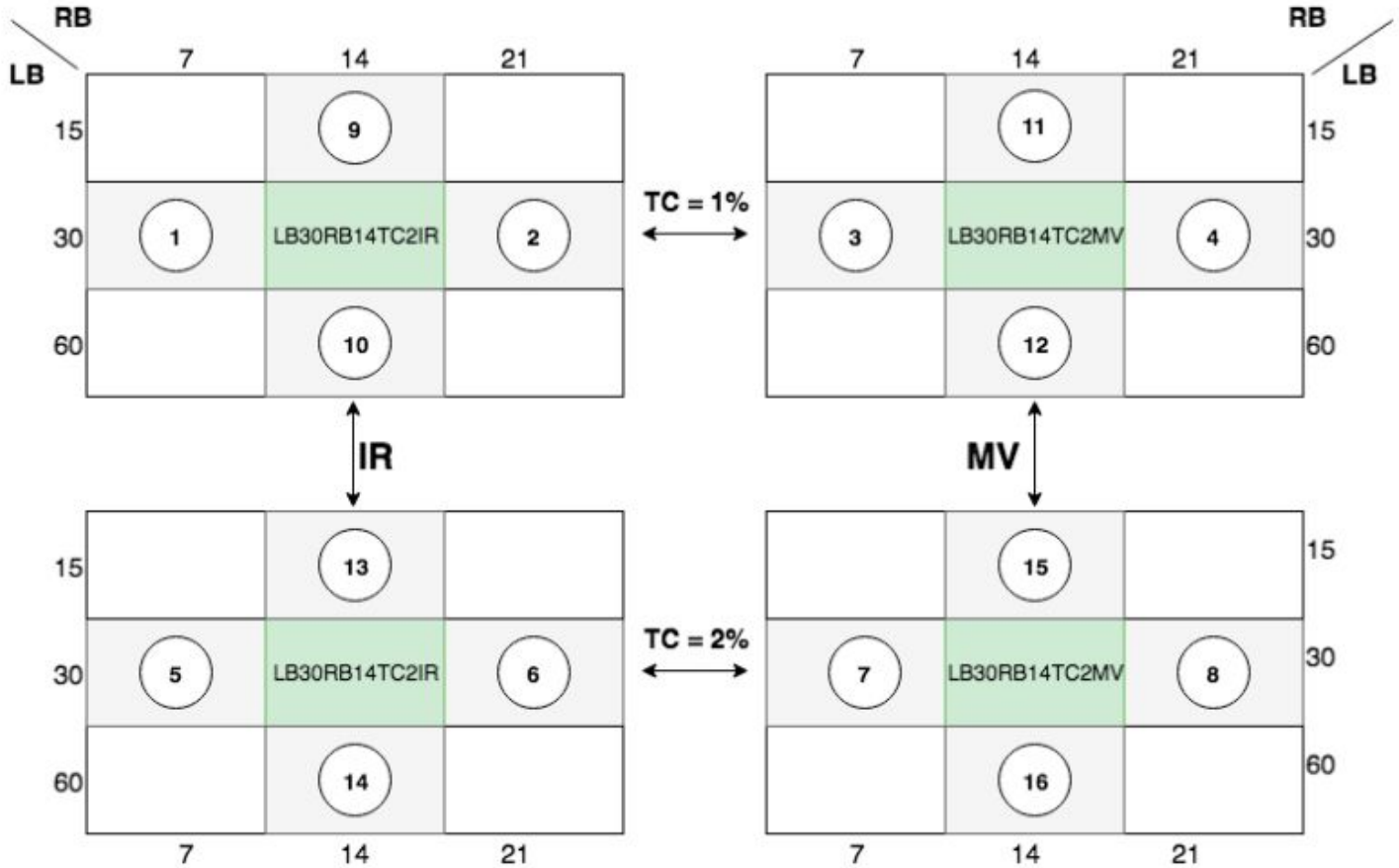
Results: Conclusions from Table1 and Figure1 (primary strategies results)

- For **LB30_RB14_TC1_IR** aRC 118.3%, much higher than any of other portfolios, especially compared to benchmarks performance, IR 1.44
- **MD is very high** for every strategy with cryptos, especially portfolios of equal weights and marketcap (passive investment)
- IR and other performance measures for Bitcoin's buy-and-hold strategy is still slightly higher than for the portfolio weighted by market capitalization, implying that **Bitcoins outperforms the crypto market substantially**
- IR for SPX (0.72) is close to the values of IR for BTC_Single (0.74), however the difference in annualized **returns** is huge **in favor of cryptocurrency**

Results: Conclusions from Table1 and Figure1 (primary strategies results)

- When **TC increased** from 1% to 2% there is almost **double decrease** in aRC for best primary strategy from the value of 118% to 64%, increase in aSD, increase in MD and almost double decrease of IR from 1.44 to 0.77 suggest that transaction costs has a strong negative impact on the strategy's profitability.
- There is no significant change of performance measures for passive investment when the TC increased from 1% to 2%

Sensitivity Analysis Parameters Combination



Source: own calculations based on period 2014/03/02 – 2019/04/26. RB – rebalancing window of strategies portfolio optimization, LB – look-back period for optimization, IR – optimization objective of maximizing Information Ratio, MV - optimization objective of minimizing variance

Table 2: Performance measures of *scenario 1:4* in sensitivity analysis, changing RB, TC=1%

	aRC	aSD	MD	IR	IRMD	IRaRCMD	nObs
MarCap10_LB30d_RB7_TC1_IR							
Markowitz	116.0	80.6	82.2	1.44	1.75	2.03	1882
EqualWeights	59.0	87.5	92.0	0.67	0.73	0.43	1882
MarkCap	50.6	72.6	88.2	0.70	0.79	0.40	1882
MarCap10_LB30d_RB21_TC1_IR							
Markowitz	215.9	94.8	80.3	2.28	2.83	6.12	1882
EqualWeights	86.7	91.0	92.2	0.95	1.03	0.90	1882
MarkCap	53.5	72.6	88.1	0.74	0.84	0.45	1882
MarCap10_LB30d_RB7_TC1_MV							
Markowitz	66.3	75.2	84.6	0.88	1.04	0.69	1882
EqualWeights	59.0	87.5	92.0	0.67	0.73	0.43	1882
MarkCap	50.6	72.6	88.2	0.70	0.79	0.40	1882
MarCap10_LB30d_RB21_TC1_MV							
Markowitz	163.3	94.5	74.6	1.73	2.32	3.78	1882
EqualWeights	86.7	91.0	92.2	0.95	1.03	0.90	1882
MarkCap	53.5	72.6	88.1	0.74	0.84	0.45	1882
BTC SPX							
BTC_Single	54.5	73.5	83.4	0.74	0.89	0.48	1882
SPX	9.5	13.2	19.8	0.72	3.62	0.34	1298

Results: Conclusions from Table2

- Markowitz strategy with RB changed from 14 to 21 days has the highest aRC (216%) and highest IR(2.28) LB30_RB21_TC1_IR
- A significant increase in aRC (163%) and IR (1.73) is shown also by the strategy LB30_RB21_TC1_MV
- The results of the Markowitz strategies for rebalancing window (RB) changed for 7 days for both objectives IR and MV (1 and 3 on Figure 11) have very similar results to strategies with the rebalancing (RB) 14 days (primary ones)
- The worst performing strategies in terms of IR is EqualWeights for RB+7 for both investment objectives IR and MV.
- In the sensitivity analysis the BTC_Single as well as SPX continue remaining stable, and in this scenario their results are very far from the Markowitz performance with LB 21 days.

Table 3: Performance measures of *scenario 5:8* in sensitivity analysis, changing RB & TC=2%

	aRC	aSD	MD	IR	IRMD	IRaRCMD	nObs
MarCap10_LB30d_RB7_TC2_IR							
Markowitz	60.3	83.6	90.5	0.72	0.80	0.48	1882
EqualWeights	54.6	87.5	92.2	0.62	0.68	0.37	1882
MarkCap	47.4	72.6	88.5	0.65	0.74	0.35	1882
MarCap10_LB30d_RB21_TC2_IR							
Markowitz	88.8	87.7	91.3	1.01	1.11	0.98	1882
EqualWeights	85.1	90.9	92.3	0.94	1.01	0.86	1882
MarkCap	52.5	72.6	88.2	0.72	0.82	0.43	1882
MarCap10_LB30d_RB7_TC2_MV							
Markowitz	50.0	81.5	88.3	0.61	0.69	0.35	1882
EqualWeights	54.6	87.5	92.2	0.62	0.68	0.37	1882
MarkCap	47.4	72.6	88.5	0.65	0.74	0.35	1882
MarCap10_LB30d_RB21_TC2_MV							
Markowitz	100.8	92.5	90.0	1.09	1.21	1.22	1882
EqualWeights	85.1	90.9	92.3	0.94	1.01	0.86	1882
MarkCap	52.5	72.6	88.2	0.72	0.82	0.43	1882
BTC SPX							
BTC_Single	54.5	73.5	83.4	0.74	0.89	0.48	1882
SPX	9.5	13.2	19.8	0.72	3.62	0.34	1298

Results: Conclusions from Table3

- The Markowitz strategies with RB 21 days outperform those with RB 7 days, however, in this case the one with objective MV shows better results LB30_RB21_TC2_MV, aRC equal to 101%, IR equal to 1.09
- There appears an unexpected observation about the fact that the strategy with RB 7 days, TC 2% and objective MV (7 on Figure 11) LB30_RB7_TC2_MV shows lower IR and higher MD than its all benchmarks including SPX.
- TC =2% negatively affects the performance of portfolios compared to primary results as well as the first scenario of sensitivity with all parameter stay the same but TC=1%

Table 4: Performance measures of *scenario 9:12* in sensitivity analysis, changing LB & TC=1%

	aRC	aSD	MD	IR	IRMD	IRaRCMD	nObs
MarCap10_LB15d_RB14_TC1_IR							
Markowitz	94.3	81.6	81.9	1.16	1.41	1.33	1882
EqualWeights	50.4	86.9	91.9	0.58	0.63	0.32	1882
MarkCap	52.4	72.6	87.9	0.72	0.82	0.43	1882
MarCap10_LB60d_RB14_TC1_IR							
Markowitz	98.1	77.9	83.2	1.26	1.51	1.49	1882
EqualWeights	50.4	86.9	91.9	0.58	0.63	0.32	1882
MarkCap	52.4	72.6	87.9	0.72	0.82	0.43	1882
MarCap10_LB15d_RB14_TC1_MV							
Markowitz	34.9	76.3	92.2	0.46	0.50	0.17	1882
EqualWeights	50.4	86.9	91.9	0.58	0.63	0.32	1882
MarkCap	52.4	72.6	87.9	0.72	0.82	0.43	1882
MarCap10_LB60d_RB14_TC1_MV							
Markowitz	94.0	76.5	80.2	1.23	1.53	1.44	1882
EqualWeights	50.4	86.9	91.9	0.58	0.63	0.32	1882
MarkCap	52.4	72.6	87.9	0.72	0.82	0.43	1882
BTC SPX							
BTC_Single	54.5	73.5	83.4	0.74	0.89	0.48	1882
SPX	9.5	13.2	19.8	0.72	3.62	0.34	1298

Results: Conclusions from Table4

- LB15_RB14_TC1_MV is the worst. The aRC – 35%, MD – 92% and IR 0.46. Much worse than all benchmarks. Huge MD and very small aRC compared to other portfolios, even majority of benchmarks, suggesting that the short look-back period does not cope with the huge variance of cryptocurrencies
- LB60_RB14_TC1_IR is the best among the group, but does not outperform best primary strategy with LB30, other parameters staying the same.
- Whole group results are worse than the first 2 groups performance

Table 5: Performance measures of *scenario 13:16* in sensitivity analysis, changing LB & TC=2%

	aRC	aSD	MD	IR	IRMD	IRaRCMD	nObs
MarCap10_LB15d_RB14_TC2_IR							
Markowitz	58.0	86.5	88.8	0.67	0.76	0.44	1882
EqualWeights	48.3	86.9	92.0	0.56	0.60	0.29	1882
MarkCap	50.7	72.6	88.1	0.70	0.79	0.40	1882
MarCap10_LB60d_RB14_TC2_IR							
Markowitz	58.2	80.8	89.4	0.72	0.81	0.47	1882
EqualWeights	48.3	86.9	92.0	0.56	0.60	0.29	1882
MarkCap	50.7	72.6	88.1	0.70	0.79	0.40	1882
MarCap10_LB15d_RB14_TC2_MV							
Markowitz	46.6	84.0	90.0	0.55	0.62	0.29	1882
EqualWeights	48.3	86.9	92.0	0.56	0.60	0.29	1882
MarkCap	50.7	72.6	88.1	0.70	0.79	0.40	1882
MarCap10_LB60d_RB14_TC2_MV							
Markowitz	61.6	79.9	87.3	0.77	0.88	0.54	1882
EqualWeights	48.3	86.9	92.0	0.56	0.60	0.29	1882
MarkCap	50.7	72.6	88.1	0.70	0.79	0.40	1882
BTC SPX							
BTC_Single	54.5	73.5	83.4	0.74	0.89	0.48	1882
SPX	9.5	13.2	19.8	0.72	3.62	0.34	1298

Results: Conclusions from Table5

- Table 5 reveals very important observations about the fact that when manipulating LB (15 and 60) under the condition of increased TC = 2%, the results of Markowitz strategies performance are very similar to their benchmarks
- The best performing strategy is LB60_RB14_TC2_MV with IR 0.77
- The worst strategy is LB15_RB14_TC2_MV with IR 0.62
- Decreasing the LB to 15 days together with increased TC =2% decreases the profitability of Markowitz portfolios significantly in comparison with primary results.

Sensitivity analysis general conclusions

- The strategy which outperforms any other in this study is Markowitz portfolio with the following parameter combination and represents the outcome of sensitivity analysis. (2) **LB30_RB21_TC1_IR** with **longer rebalancing period** than initially assumed and the strategy aiming at **maximizing risk-adjusted return** through setting objective of max IR.
- The results are generally also improved when the **rebalancing period** is **increased** from 14 to 21 days.
- Increase of **look-back** period from 30 to 60 has ambiguous impact on Markowitz portfolios
- **Transaction costs** increase have **negative impact** on performance of Markowitz strategies and decrease the excess return of this strategies over benchmarks.
- There are yet many **other parameters** variation to be tested and more conclusions to be drawn from this research.

Thank you!

- Working Paper: https://www.wne.uw.edu.pl/files/5016/1041/4810/WNE_WP347.pdf
- Application: https://www.wne.uw.edu.pl/sakowski/cryptoMarkowitz.html
- Contact: pawelsakowski@gmail.com, annaturovsteva20@gmail.com

Appendix

Portfolio returns calculations

$$R_{0,T}^{(p)} = \prod_{t=1}^{T}\left(1 + \sum_{i=1}^{N} w_i r_{i,t} - \Delta W_t^R * TC\right) - 1$$

Portfolio returns are obtained by multiplying cryptocurrencies returns by the lagged values of their weights at each RB date.

For the periods between rebalancing dates, weights of assets are adjusted in relation with their returns.

$$\Delta W_t^R = \frac{\sum_{i=1}^{N} |x_{i,t}|}{PV_t}$$

$$x_{i,t} = w_{i,t} - w_{i,t-1}$$

Formula for equity line:

$eq.line_t = (1 + r_1)(1 + r_2) \dots (1 + r_t)$, where r_t is a simple daily return.

$$w_{i,t-1} = w_{i,t-2} * (1 + r_{i,t-1})$$

N – number of assets, $i \in \{1, \dots, N\}$,

T – total number of intervals (duration of the investment) in the period between 0 and T, $t \in \{1, \dots, T\}$,

$R_{0,T}^{(p)}$ – simple rate of return of the portfolio in the period between 0 to T, $R_{0,T} = \frac{P_T - P_0}{P_0}$

$w_{i,t}$ – percent weight of i-th asset in the whole portfolio in the period of t,

$r_{i,t}$ – rate of return of i-th asset in the period t,

ΔW_t^R – portfolio allocation change (range from 0% up to 100%) between day t -1 and day t, where day t is the reallocation day.

TC – transactional costs at the level of 1% or 2%,

$x_{i,t}$ – allocation change (range from 0% up to 100%) for i-th crypto between day t -1 and day t, where day t is the reallocation day,

$MC_{i,t}$ – market cap of the i-th asset on day t,

MC_t – sum of all market caps on day t.

$$w_{i,t} = \frac{(1 + r_{i,t}) * w_{i,t-1}}{1 + \sum w_{i,t-1} * r_{i,t}} - Markowitz\ rebalancing$$

$$w_{i,t} = \frac{1}{N} = \frac{1}{10} - equally\ weightes\ rebalancing$$

$$w_{i,t} = \frac{MC_{i,t}}{\sum_i^N MC_{i,t}} - market\ cap.weighted\ rebalancing$$