



Replicating mutual funds strategies using linear regression models

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Agenda

- Introduction
- Research Hypothesis
- Methodology
- Data
- Results
- Conclusions

Introduction

- Mutual fund - investment vehicle, that takes money from many investors to purchase securities.
- Regulated by government, must be registered by Securities and Exchange Commission.
- First mutual fund in Netherlands(1870)
- Over 15 000 funds in USA (2010)
- Worldwide assets 24,7 trillion dollars

Introduction

Mutual Funds

Advantages	Disadvantages
Diversifications	Fees and expenses
Regulation	Trading limitation
Professional management	Loss of control
Low investment minimum	Inefficiency of cash reserves
Convenience	Size

Main Mutual Funds Types

Money Market	Bond/Income	Equity	Speciality	Index Funds
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Research Hypothesis

The selected derivatives contracts traded on Eurex can replicate well performing world allocation mutual funds strategies.

Methodology (Linear regression)

- Linear regression model

$$y_i = \beta_1 * x_{i1} + \dots + \beta_p * x_{ip} + \varepsilon_i$$

y_i = fund daily return of i day

x_i = daily return of p asset in i day

e_i = error term

$$x_i = \frac{P_i}{P_{i-1}} - 1 \quad \hat{\sigma}_{\varepsilon}^2 = \frac{SSE}{N - p} \quad SSE = \sum_{i=1}^N \varepsilon_i^2$$

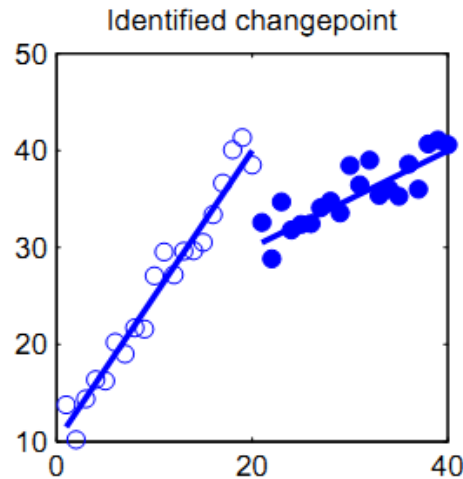
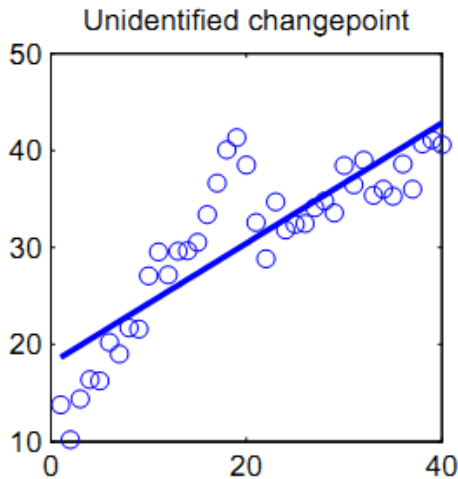
$\hat{\sigma}_{\varepsilon}^2$ = mean square error

p = number of assets

N = number of observation

Methodology (Structural Break Tests)

- Change point – sudden, unexpected change to underlying relationships generating the data.
- Identification methods: Chow and Quandt's log likelihood ratios.



$$Y = XB_1 + E, \quad \text{when } T < t$$

$$Y = XB_2 + E, \quad \text{when } T > t$$

Methodology (Chow-test)

- Null hypothesis : $B_1 = B_2$
- Alternative hypothesis: $B_1 \neq B_2$
- RSS -residual sum of whole sample, T obs.
- RSS1 – residual sum of first sub-sample, m obs.
- RSS2 - residual sum of second sub-sample, n obs.
- k – number of explanatory variables in sub-sample

Methodology (Chow-test)

- F-test for testing H_0 :

$$F(k, T - 2k) \approx \frac{(RSS - (RSS1 + RSS2))(T - 2k)}{(RSS1 + RSS2)k}$$

- $T = m + n$, total number of observation
- $2k =$ number of independent variables in regression on whole sample

Methodology (Quandt's-test)

- Quandt's log likelihood ratio

$$\lambda = \log \frac{\sigma^t \sigma_2^{(T-t)}}{\sigma^T}, \quad t = 1, \dots, T$$

where

- σ is the standard error of estimate for a regression taking into account all observations
- σ_1, σ_2 are the standard error of estimate for a regression spanning the first and the second sub-sample, respectively
- T is the number of observations in the whole sample.
- t is the number of observations in the first sub-sample.

- Examines likelihood function
- Null hypothesis : $B_1 = B_2$

Algorithm

- 1. For each max. number = {3,5,7} of futures contracts in portfolio
- 2. Main Loop (Step=10)
 - Initial window = 250
 - 3. For given initial window
 - Make regressions to find optimal portfolio structure
 - Lowest MSE means best portfolio
 - For selected portfolio
 - Check structure stability
 - If structure is stable -> make forecast, make Step and go to (2)
 - Else shorten initial window on left by 10 and go to (3)

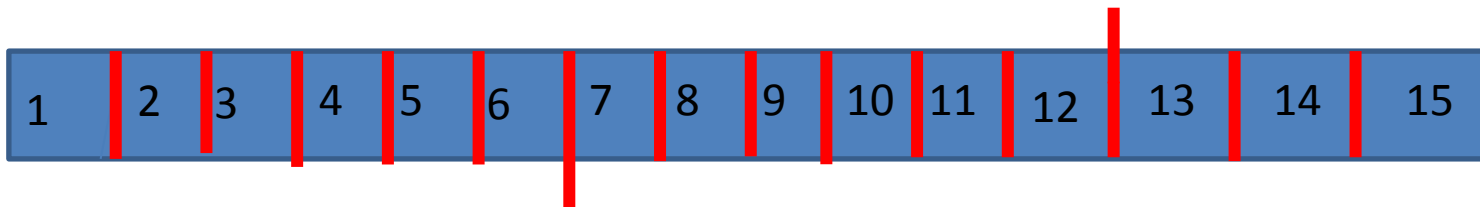
Algorithm

1. Max number of products in portfolio = 3

2. Main Loop Initial window = 12

3. For given initial window

Find best structure of portfolio (add futures one by one)



4. For best portfolio

if structure of subsample(1-12) stable

make forecast for 13,14 and go to (2),

we will operate sample (3-15)

else

cut 2 observation from left and go to (3)

we will operate on (3-12) sample

Data

- Period : 12/2008- 02/2011
- Eurex Exchange Products : Equity Futures and Index Futures (50 products)
- Best performing mutual funds:
 - UBS Global Frontier (BGFAX)
 - BlackRock Global Allocation Inv. (MDLOX)
 - Ivy Asset Strategy Fund (WASAX)
 - First Eagle Global Fund (SGENX)
- Sources : moneycentral.msn.com

Data (UBS Global Frontier)

- Long term returns on capital
- Invest directly/indirectly in equity ,fixed income securities
- Also interest future contracts and synthetic UBS products
- Low diversification
- (BGFAX)

Data (BlackRock Global Allocation)

- Open-end fund (USA)
- Global equity, debt, money market securities
- Highly diversified
- (MDLOX)

Data(Ivy Asset Strategy Fund)

- Open-end fund (USA)
- Stock ,bonds, short term investments
- (WASAX)

Data (First Eagle Global Fund)

- Open-end (USA)
- equities and bonds
- Highly diversified
- (SGENX)

Fees and expenses

Table 1: Fees and Expenses in %

	BGFAX	MDLOX	WASAX	SGENX
Front Load	5.50	5.25	5.75	5.00
Back Load	0.00	0.00	1.00	0.00
Current Mgmt Fee	0.95	0.75	0.56	0.75
Redemption Fee	1.00	0.00	2.00	2.00
12b1 Fee	0.25	0.75	0.25	0.25
Expense Ratio	1.40	1.06	0.97	1.13
Aprox. 3-year total cost	9.70	8.43	8.66	8.39

Source: Bloomberg.com

Results(Aggregated)

Results (BGFAX)

Table 2: BGFAX Results - Mean Absolute Error and Average Window Length

MAE / Number of instruments in portfolio	3	5	7
Quandt test	0.00892	0.00916	0.00939
Chow test	0.00897	0.00897	0.00928
Fixed window 50	0.00940	0.00965	0.01015
Fixed window 100	0.00911	0.00964	0.00988
Fixed window 150	0.00902	0.00929	0.00951
Fixed window 200	0.00877	0.00885	0.00912
Fixed window 250	0.00874	0.00890	0.00919
Average Window Length / Number of instruments in portfolio	3	5	7
Quandt test	181.96	186.07	201.79
Chow test	223.04	222.14	223.39
Fixed window 50	50.00	50.00	50.00
Fixed window 100	100.00	100.00	100.00
Fixed window 150	150.00	150.00	150.00
Fixed window 200	200.00	200.00	200.00
Fixed window 250	250.00	250.00	250.00

Results(BGFAX)



Results(MDLOX)

Table 3: MDLOX Results - Mean Absolute Error and Average Window Length

MAE / Number of instruments in porfolio	3	5	7
Quandt test	0.00429	0.00439	0.00446
Chow test	0.00424	0.00427	0.00455
Fixed window 50	0.00457	0.00493	0.00530
Fixed window 100	0.00446	0.00449	0.00457
Fixed window 150	0.00438	0.00452	0.00456
Fixed window 200	0.00423	0.00427	0.00430
Fixed window 250	0.00416	0.00424	0.00432
Average Window Length / Number of instruments in porfolio	3	5	7
Quandt test	180.54	196.61	202.68
Chow test	221.43	230.54	223.75
Fixed window 50	50.00	50.00	50.00
Fixed window 100	100.00	100.00	100.00
Fixed window 150	150.00	150.00	150.00
Fixed window 200	200.00	200.00	200.00
Fixed window 250	250.00	250.00	250.00

Results(MDLOX)

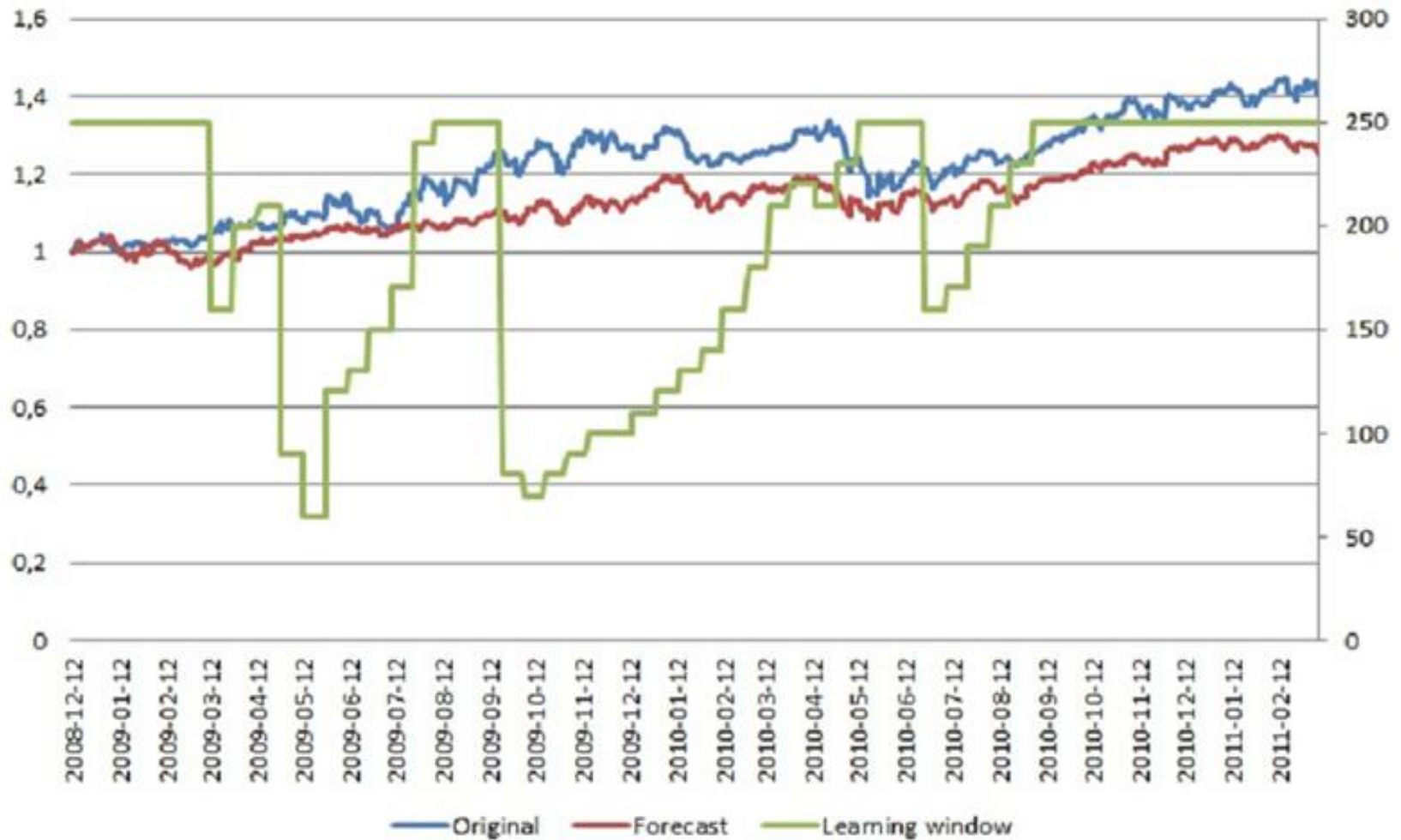


Results(WASAX)

Table 4: WASAX Results - Mean Absolute Error and Average Window Length

MAE / Number of instruments in porfolio	3	5	7
Quandt test	0.00613	0.00621	0.00641
Chow test	0.00583	0.00584	0.00583
Fixed window 50	0.00617	0.00672	0.00718
Fixed window 100	0.00607	0.00619	0.00650
Fixed window 150	0.00609	0.00622	0.00640
Fixed window 200	0.00611	0.00618	0.00615
Fixed window 250	0.00598	0.00596	0.00597
Average Window Length / Number of assets in porfolio	3	5	7
Quandt test	163.04	167.50	172.32
Chow test	195.89	207.50	207.32
Fixed window 50	50.00	50.00	50.00
Fixed window 100	100.00	100.00	100.00
Fixed window 150	150.00	150.00	150.00
Fixed window 200	200.00	200.00	200.00
Fixed window 250	250.00	250.00	250.00

Results(WASAX)

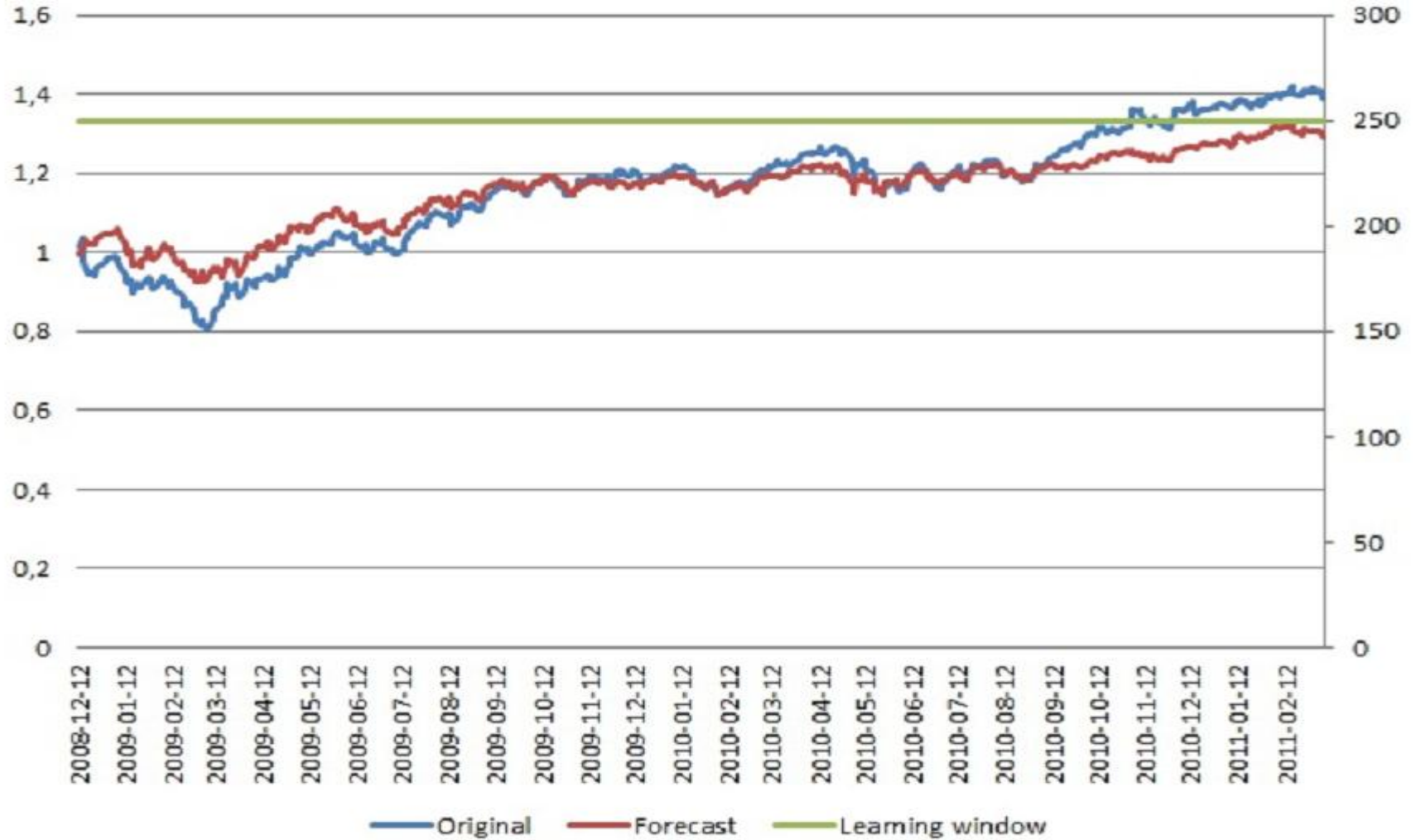


Results (SGENX)

Table 5: SGENX Results - Mean Absolute Error and Average Window Length

MAE / Number of instruments in portfolio	3	5	7
Quandt test	0.00460	0.00473	0.00480
Chow test	0.00477	0.00489	0.00487
Fixed window 50	0.00506	0.00522	0.00573
Fixed window 100	0.00458	0.00471	0.00488
Fixed window 150	0.00465	0.00490	0.00505
Fixed window 200	0.00456	0.00470	0.00481
Fixed window 250	0.00454	0.00460	0.00479
Average Window Length / Number of assets in portfolio	3	5	7
Quandt test	196.25	197.50	202.50
Chow test	220.54	222.14	227.68
Fixed window 50	50.00	50.00	50.00
Fixed window 100	100.00	100.00	100.00
Fixed window 150	150.00	150.00	150.00
Fixed window 200	200.00	200.00	200.00
Fixed window 250	250.00	250.00	250.00

Results (SGENX)



Conclusions

- Only in one case (WASAX) Chow and Quandt tests enable to achieve the lowest replication error
- MAE is not appropriate measure of overall replication performance
- Portfolio consists of 3 assets yields better results than portfolio consists of 5 and 7 assets
- Eurex products could be useful tools in replicating the most successful mutual funds strategies, however there is need to calculate accurate transactions costs.

Futher Research

Futher Research

Thank You